NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

B. COMM ACTUARIAL SCIENCE PART II

APRIL /MAY 2003 EXAMINATIONS

SURVIVAL MODELS : CIN 2212

TIME ALLOWED: 3 HOURS

INSTRUCTIONS TO CANDIDATES

1. Attempt all 5 questions, beginning your answer to each question on a separate sheet.
2. Mark allocations are shown in brackets.

Additional Examination Material

Non-programmable electronic calculator
Tables for Actuarial Examinations (Actuarial Tables)

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1. The forces of mortality at age x under three different modifications of a standard table, μx are given by the following:

   (i) $\overset{1}{\mu x} = \mu n + \kappa$ κ>0

   (ii) $\overset{ii}{\mu x} = c.\mu x$ c>1

   (iii) $\overset{iii}{\mu x} = \mu x + r$ $r > 1$

   for χ where μn is a monotonic increasing function of age.

   (a) For each of the above modifications, find tPx^{mod}, the modified probability of survival and compare it with tPx, the unmodified mortality.

   (9 marks)

   (b) What are the effects of the above modifications on the life expectancy of the lives represented by the new mortality.

   (2 marks)

   (c) Under condition (ii) with c = 3 find the present value of the benefits of a 20 year pure endowment with sum assured of $50,000 bought by a life aged 35. Assume a standard mortality of AM92 Ultimate at an effective interest of 15.75%.

   (5 marks)

   (Total marks 16)

2. Let X be the present value random variable defined by

$$X = \begin{cases} a_{\overline{T_x}|} & 0 \le T_x < n \\ \bar{a}_{\overline{n}|} & T_x \ge n \end{cases}$$

a) Explain what X represents and give the actuarial notation for E(X).

[2 marks]

b) Prove that $\bar{A}_{x:n} + \delta \bar{a}_{x:n} = 1$ **[5 marks]**

c) Prove that $V_{ar}(X) = \frac{2}{\delta}\left[\bar{a}_{x:n} - {}^2\bar{a}_{x:n}\right] - \bar{a}_{x:n}^2$ for the random variable X defined above.

[8 marks]

[Total 15 marks]

3. The usual mortality assumptions for mortality between integral ages x and x + 1 are:

Uniform Distribution of deaths ${}_tq_x = t.q_x$

The Balducci assumption ${}_{1+t}q_{x+t} = (1 - t).q_x$

Constant force of mortality ${}_tq_x = 1 - e^{-③t}$ all for 0 [t [1

(a) Under the Balducci assumption show that:

$${}_tP_x = \frac{P_x}{1-(1-t).q_x}$$

and hence or otherwise show that

$${}_yq_{x+t} = \frac{y.q_x}{1-(1-y-t).q_x} \qquad \text{for } 0 [\ t < 1 \qquad 0 \le y < 1$$

$$y + t \le 1$$

(b) Find ${}_4p_{25\frac{3}{4}}$ assuming the Balducci hypothesis and using the AM92 Ultimate mortality.

(5 marks)

(c) Draw graphs of μ_x between ages x and x + 1 for each of the three above assumptions. The graphs should show clearly the movement of μ_{x+t} for 0 [t [1 and should be on one diagram. **[3 marks]**

(d) Assuming that a certain group of lives was observed between ages x and x + 1 and the following information was collected:

A = number of lives under observation at exact age x
N = new entrants at age x + r
W = withdrawals at age x + s $\quad r < s$
B = number remaining at age x + 1
D = number of deaths observes for those lives under observation.

For each of the above mortality assumptions find an expression of D in terms of q_x and demonstrate why the Baducci assumption is better under these circumstances. **[5 marks]**

[Total 38 marks]

4. A man aged 40 years buys a deferred annuity of $25 0000 per annum payable in advance from age 65. A death benefit of $200 000 is payable immediately in the event of earlier death.

(i) Calculate the level annual premium payable. **[6 marks]**

(ii) If the benefit is altered so that the annuity is paid for a guaranteed period of 10 years, find the new premium, for a new contract similar to the one above. **[5 marks]**

(iii) Assuming that the man dies during the deferred period and that the death benefit is used to buy an annuity for the spouse now aged 50, calculate the annual annuity received by the spouse, if it has a 5 year guarantee period with the first payment due now.

[4 marks]

Basis: Mortality: AM92 Ultimate before age 65 [for male life]
PMA92 C20 after age 65 [for male life]
PFA92 C20 for female annuitant.

Interest: 40% per annum effective

Expenses are ignored.

[Total 15 marks]

5. In a study to compare the failure rate of two light bulbs made by two competitors, the following information concerning the life times of 10 of each types was gathered. The experiment was stopped after 20 weeks

Star bright Light Bulbs	
Bulb number	Life time (weeks)
1	17
2	Did not fail by end of experiment
3	19
4	Did not fail but was stolen after 15 weeks
5	17
6	Did not fail by end of experiment
7	5
8	14
9	Did not fail but was accidentally removed after 12 weeks
10	18

Silver Minor Bulbs	
Bulb number	Life time (weeks)
1	15
2	9
3	Did not fail by end of experiment
4	6
5	Did not fail but was stolen after 11 weeks
6	15
7	Did not fail but was stolen after 13 weeks
8	7
9	12
10	Did not fail but was removed accidentally after 10 weeks

(i) Calculate the Kaplan-Meier estimate $\hat{F}_{(t)}$ of the distribution function $F_{(t)}$ for both Starbright **[5 marks]**
and Silver Mirror light bulbs. **[5 marks]**

(ii) Estimate the probabilities that a bulk will fail before the end of 15 weeks, separately for both Starbright and Si9lver Mirror and compare the results.
[1 mark]
[1 mark]

(iii) Calculate the variance of $F_{(t)}$ at time t when each observation is made, separately for the two bulb types. **[3 marks]**
[3 marks]

(iv) Using any of the results above, compare the lives of the given bulbs.

[2 marks]
[Total 20 marks]

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