NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

B.COMM (HONOURS) DEGREE IN ACTUARIAL SCIENCE

SURVIVAL MODELS (CIN 2212)

APRIL / MAY 2006 SECOND SEMESTER EXAMINATIONS

TIME ALLOWED : 3 HOURS

Instructions to Candidates

1. Answer **ALL 12** questions.

Requirements

1. Actuarial Tables Both Yellow Book and Green Book.
2. Non Programmable Scientific calculator.

Question 1

The random variable K represents the curtate future lifetime of a person now aged x. Define A_x at the rate of interest ' i ' to be $E[V^{K+1}]$.

(i) Derive an expression in terms of assurance functions for the variance of V^{K+1}.

[2 marks]

(ii) Derive an expression in terms of assurance functions for the expected value and variance of $\ddot{a}_{\overline{K+1}|}$

[3 marks]

[Total: 5 marks]

Question 2

(i) T_x denotes the future lifetime of a life currently aged x. Write down the p.d.f of T_x. **[1 mark]**

(ii) Using your answer to (i), show that:

(a) $\frac{d}{ds} \log {}_sP_x = -\mu_{x+s}$, and

(b) ${}_tP_x = \exp\{-\int_0^t \mu_{x+s}\, ds\}$ **[4 marks]**

(iii) In a certain population, the force of mortality is given by:

	μ_x
$60<x\leq70$	0,01
$70<x\leq80$	0,015
$x>80$	0,025

Calculate the probability that a life aged exactly 65 will die between exact ages 80 and 83. **[3 marks]**

[Total: 8 marks]

Question 3

A life aged 40 effects a 20 – year endowment policy with basic sum assured $100 000, payable at the end of the year of death if death occurs within 20 years or on survival to the end of the term. The policy has annual premiums payable for at most 15 years and calculated on the basis of AM92 Ultimate mortality with 4% per annum interest.

(i) Calculate the annual premium. **[3 marks]**

(ii) Write down an expression for the prospective reserve at duration 3 years. Evaluate your expression. **[4 marks]**

(iii) Write down an expression for the retrospective reserve at duration 3 years. Evaluate your expression. **[4 marks]**

(iv) State the conditions for which the prospective reserve equals retrospective reserve. **[3 marks]**

[Total: 14 marks]

Question 4

(a) List the data required for the exact calculation of the central exposed to risk of lives aged x last birthday in a mortality investigation over the two- year period from 01 January 2001 to 01 January 2003.

[3 marks]

(b) In an investigation of mortality during the period 01 January 2001 to 01 January 2003, data are available on the number of lives under observation, aged x last birthday, on 01 January 2001, 01 July 2001 and 01 January 2003.

Derive an approximation for the central exposed to risk at age x last birthday over the period in terms of the populations recorded on each of these three dates.

[3 marks]

(c) A life insurance company has carried out an investigation, over N years, of the mortality of its term assurance policyholders. Premiums for these contracts are based on the policyholder's age last birthday at the date of issue.
The following data are available from the investigation:

d_x = number of deaths during the investigation aged x

$P_{x,t}$ = number of lives under observation aged x at time t (t=0, 1, ...,N)

where x is the policyholder's age last birthday at date of issue plus the number of policy anniversaries passed.

(i) State the type of rate interval. **[1 mark]**

(ii) Write down an expression that may be used to evaluate the central exposed to risk using the available data for P. State any assumptions made. **[3 marks]**

[Total: 10 marks]

Question 5

During a 2 year trial for a new medical treatment, 50 patients were observed after receiving the new treatment on 1 July 2000. For those patients who died or who left the trial before 30 June 2002, a record was kept on their time spent under observation. The details are shown bellow:

Period under investigation (in months) for patients who:

Died	Left trial
4	2
6	5
6	7
8	9
11	10
16	13
22	15
	18

(i) Calculate the Kaplan – Meier estimate of the survival function, *S(t)*. **[6 marks]**

(ii) Sketch the hazard, *h(t)*, implied by the Kaplan- Meier estimate of *S(t)*. **[3 marks]**

[Total: 9 marks]

Question 6

Show that, under the assumption of the two -state Markov model for mortality:

$$\text{Var}[Di - \mu Vi] = E[Di].$$ Note: you are given that $E[Di - \mu Vi] = 0$

[Total: 6 marks]

Question 7

A particularly morbid actuarial student is considering the mortality of the population of the village in which she lives. The village graveyard contains tombstones which bear inscriptions of the form " A N Other beloved mother of ….., Born 1903, Died 1976".

(i) If the students were to use the inscriptions as a source of death data, state the exact age to which the calculated mortality rate would apply assuming a Binomial model was used. **[2 marks]**

(ii) Comment on the difficulties the student is likely to face in attempting to calculate mortality rates in this way. **[3 marks]**

[Total: 5 marks]

Question 8

An investigation into the mortality of young adult males in a developed country has been undertaken. The table below shows an extract from the results.

Age	Exposed-to-risk	Observed deaths	Standardised deviation Z_x
18	34,000	40	1.9159
19	33,000	35	1.4541
20	29,500	27	0.4460
21	30,000	26	0.0394
22	25,500	22	-0.1469
23	24,000	19	-0.5106
24	17,000	13	-0.5067
25	23,500	20	-0.0467
26	18,000	12	-0.8437
27	14,000	11	-0.2609

(i) Someone suggests to you that the underlying mortality of these young men is the same as that in English Life Table 15 (Males) and that you should test this using the chi-squared test.

(a) Define the standardized deviation.
(b) State the null hypothesis to be tested.
(c) State the test statistic and its distribution under the null hypothesis.
(d) Carry out the test.

[8 marks]

(ii) Explain how you would graduate the observed experience by reference to a standard mortality table. **[5 marks]**

[Total: 13 marks]

Question 9

Let T_0 be a random variable denoting the future lifetime of a newly born child and T_x be the random variable denoting the future lifetime of a life aged x. Define $S_0(x) = P(T_0 > x)$ and $S_x(t) = P(T_x > t)$.

(a) Show that $\frac{d}{dt}\,{}_tP_x = -\,{}_tP_x\,\mu_{x+t}$ **[3 marks]**

(b) Find the probability distribution of T_x **[2 marks]**

(c) Show that $E[T_x] = \int_0^\infty {}_tP_x\,dt$ **[2 marks]**

(d) By defining K_x as the random variable denoting the integer part of T_x, show that $E[K_x] \approx E[T_x] - 0.5$ **[3 marks]**

[Total: 10 marks]

Question 10

During a period of length T years, you observe a total of N lives between the ages of x and x + 1. You do not necessarily observe each life for the entire year of age. The total time spent under observation by the N lives is V. d deaths are observed.

(i) State the assumptions underlying the Poisson model for d, given that the force of mortality between the ages of x and x + 1 is a constant, μ. **[2 marks]**

(ii) Show that the maximum likelihood estimator of μ is D / V, under the assumptions in (i) above. **[2 marks]**

(ii) Show that the maximum likelihood estimator has:

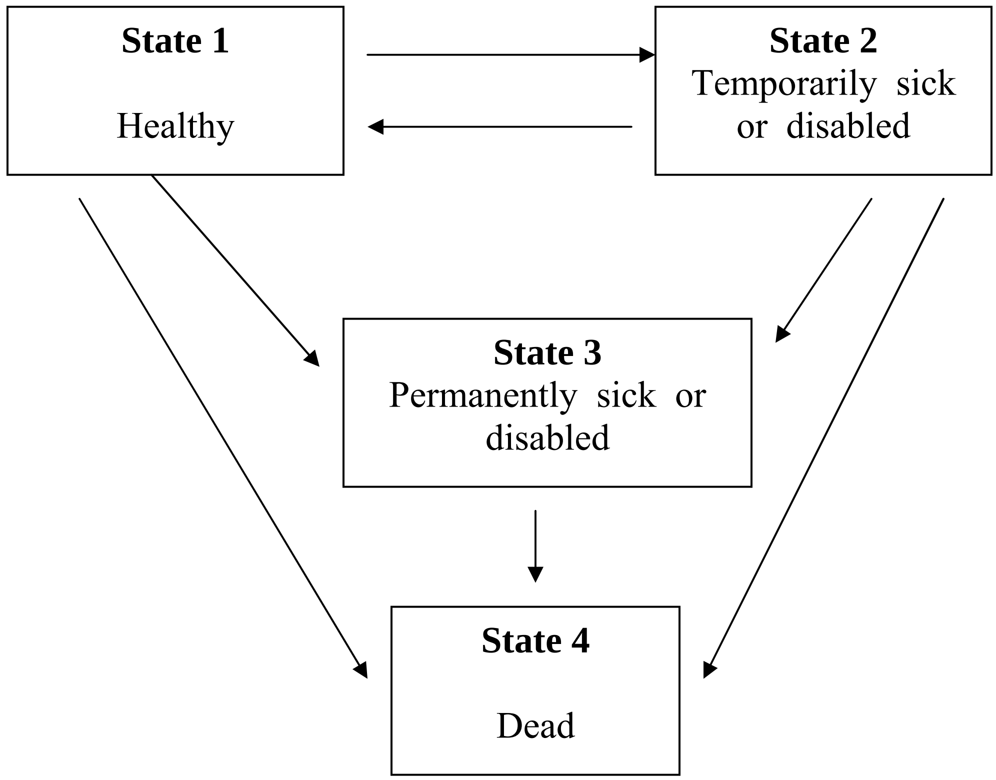
(a) an expected value of μ **[1 mark]**

(b) a variance of μ / V **[1 mark]**

[Total: 6 marks]

Question 11

The following diagram represents a four – state Markov model.



The force of transition from state i to state j ($i \neq j$) at age x is denoted by μ_x^{ij} and the probability that a life, who is in state i when aged x, will be in state j at age x + t is ${}_t\mathrm{P}_x^{ij}$.

(i) Derive from first principles a differential equation for ${}_t\mathrm{P}_x^{23}$, stating all assumptions made. **[5 marks]**

(ii) Given that, for x = 40, 41:
${}_1\mathrm{P}_x^{12} = 0.03$, ${}_1\mathrm{P}_x^{13} = 0.002$, ${}_1\mathrm{P}_x^{14} = 0.001$, ${}_1\mathrm{P}_x^{21} = 0.4$, ${}_1\mathrm{P}_x^{23} = 0.1$, ${}_1\mathrm{P}_x^{24} = 0.01$ and ${}_1\mathrm{P}_x^{34} = 0.3$,
Calculate, ${}_2\mathrm{P}_{40}^{13}$. **[2 marks]**

(iii) An insurance company issues a combined sickness, disability and assurance contract that provides the following benefits:

- An income payable while the policyholder is temporarily sick or disabled;

- A lump sum payable either on becoming permanently sick or disabled, or on death.

The contract terminates as soon as the lump sum has been paid.

Explain how the model could be simplified for the purpose of modeling the claims process involved. State how your answer to (i) would be altered as a result of this change. (You are not required to derive this result from first principles). **[2 marks]**

[Total: 9 marks]

Question 12

(a) Define the following terms:

(i) Type 1 and Type 11 censoring. **[1 mark]**

(ii) Left censoring and right censoring. **[1 mark]**

(b) Explain why it is necessary to graduate crude rates of mortality for practical use.

[3 marks]

[Total: 5 marks]

END OF EXAMINATION PAPER!!!