

**NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY**

**B. COMM (HONOURS) DEGREE ACTUARIAL SCIENCE**

**ACTUARIAL MATHEMATICS – CIN 4010**

**APRIL/MAY 2003 EXAMINATION**

**DURATION : 3 HOURS**

**INSTRUCTIONS TO CANDIDATES**

1. Answer all questions.
  2. You must not start writing your answers in the booklet until instructed to do so by the invigilator
  3. Answer to each question on a separate sheet.
  4. In addition to this question paper you should have an electronic calculator and actuarial tables
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**Question 1**

Consider a multiple decrement model with 3 decrements . You are given that the one year single decrement rates of decrement are equal to  $q_x^\alpha = 0.003$ , and  $q_x^\beta = 0.24$ , and  $q_x^\gamma = 0.05$ , respectively

- (a) Under assumption of a uniform distribution of decrements over the year of age  $(x, x + 1)$  in each single decrement model, calculate the dependent rate of decrement  $(aq)_x^\alpha$ . **[2 marks]**
- (b) Under the assumption of a constant force of decrement over the year of age  $(x, x + 1)$  in each single decrement model, calculate the dependent rate of decrement  $(aq)_x^\alpha$  **[3 marks]**

**[Total 5 marks]**

**Question 2**

- (a) Explain the term “deferred period” in the content of a permanent health insurance policy. **[2 marks]**
- (b)
  - (i) Explain the term “off period” in the context of a sickness policy.
  - (ii) Explain what would be the effect of changing the off period for a sickness policy which provides a level sickness benefit for all durations of sickness. **[4 marks]**

**[Total 6 marks]**

### **Question 3**

- (i) Describe how option pricing techniques may be used to determine the value of the guarantee under a deferred annuity policy with a guaranteed minimum annuity. **[2 marks]**
- (ii) Define each of the following terms and given an example of each in life assurance business:
- (a) class selection
  - (b) spurious selection
  - (c) adverse selection

**[3 marks]**

### **Question 4**

A pension scheme provides an ill-health retirement pension of  $\frac{1}{60}$  of Final Pensionable Salary for each year of company service, with fractions of a year to count proportionately, subject to a maximum pension of  $\frac{40}{60}$  of Final Pensionable Salary. Retirement due to ill-health may take place at any age before age 65. Final Pensionable Salary is defined as the average annual salary over the three-year period preceding retirement.

Derive commutation functions to value the ill-health retirement pension for a member aged exactly 25, who has completed exactly 5 years company service to date. Define carefully all the symbols that you use. **[7 marks]**

### **Question 5**

An insurer issues 10 year endowment assurance policies to lives aged 60 who have provided satisfactory answers on a basic medical questionnaire. The policy includes an option at maturity that allows policyholders to take out a whole life policy based on the same sum assured (payable at the end of the year of death) and standard premium rates, irrespective of their state of health at that time.

The insurer calculates standard premiums using the A1967/70 Select mortality table and 4% interest, with an expense allowance of 5% of all premiums. Assuming that the mortality of those policyholders who exercise the option can be modelled using ultimate rates and that all surviving policyholders will exercise the option, what extra premium should be insurer charge in addition to the basic endowment assurance premium for a policy with a sum assured of £25,000 to allow for this option?

[You should assume that the extra premium is payable during the term of the original policy].

**[Total 7 marks]**

### **Question 6**

Given the following multiple decrement table, calculate the single decrement rates  $q_{45}^{\alpha}$ ,  $q_{45}^{\beta}$ ,  $q_{46}^{\alpha}$  and  $q_{46}^{\beta}$ , stating carefully all assumptions you make.

Age x	(a1) <sub>x</sub>	(ad) <sub>x</sub> <sup>α</sup>	(ad) <sub>x</sub> <sup>β</sup>
45	10,000	26	154
46	9,820	30	170
47	9,620		

**[Total 8 marks]**

### **Question 7**

A husband and a wife, aged 64 and 60, respectively, buy a life insurance policy, which provides a reversionary annuity of \$15,000 p.a., payable monthly in arrears throughout the lifetime of the wife after death of husband, provided that the husband dies within 10 years of the start date of the policy.

Calculate the gross single premium for this contract on the following assumptions:

Mortality: a(55) ultimate for males and females as appropriate;  
Interest: 4% per annum  
Expenses: Initial expenses of £50 and 3% of the gross single premium;  
Annuity expenses of \$2 per annuity payment.

**[Total 8 marks]**

### **Question 8**

Members of a friendly society pay a level weekly contribution which ceases at age 65 and is waived during periods of sickness. The benefits are as follows:

- a sickness benefit of £50 per week for the first 26 weeks of sickness, £35 per week for the next 26 weeks and £20 per week thereafter, the benefit ceasing at age 65;
- immediately on death before age 65, a lump sum of \$10,000;
- on survival to age 65, an annuity of \$3 000 per annum payable by monthly instalments in advance.

There is no waiting period for the sickness benefit and the off period may be assumed to be the same at the underlying the tables in Formulae and Tables for Actuarial Examinations.

Calculate the weekly contribution payable by a male aged 30 exactly at entry.

Basis:

Mortality before age 65: English Life Tables No. 12 Males

Mortality after age 65: a(55) Males (ultimate mortality)

Sickness: Manchester Unity Sickness Experience 1893-97,  
Occupation Group A H J

Interest: 4% per annum.

**[Total 10 marks]**

### **Question 9**

A life office issues a proposer aged 40 a 20 year with profits year endowment assurance policy for a sum assured of £10,000. The sum assured and reversionary bonuses are payable at the end of the year of death.

Calculate the annual premium payable annually in advance throughout the term of the contract, if the office assumes that future reversionary bonuses will be at a rate of 1.94175% of the sum assured, compounded annually and vesting at the start of the policy year.

Basis:

Mortality: A1967-70 Select

Interest: 5% per annum.

Initial expenses: £60 + 1% of the basic sum assured + 10% of the first premium

Renewal expenses: 2% of each premium excluding the first.

Claim expenses: £50, applying to death benefits only.

**[Total 12 marks]**

### **Question 10**

A Life insurance company issues with-profit whole of life assurance contracts in accordance with the following premium basis:

Mortality: A1967-70 Ultimate.

Interest: 4% per annum.

Initial expenses: £80 + 70% of the first gross monthly premium

Renewal expenses: 3% of each monthly premium, excluding the first.

Reversionary bonuses: 2.5 per annum simple, vesting at the end of each year.

On death the sum assured, plus all bonuses declared up to the date of death, are assumed to be payable at the end of the year of death.

A policyholder then aged 40 exact at entry took out such a policy on 1<sup>st</sup> January 1992, with a basic sum assured of £10,000. Premiums are paid monthly in advance.

- (a) Calculate the gross monthly premium. **[10 marks]**
- (b) On 1<sup>st</sup> January 2002, the policy had £3,000 of bonuses attached to it. Calculate the gross premium prospective policy value on 1<sup>st</sup> January 2002, assuming that the policy valuation basis is the same at the premium basis.

**[5 marks]**  
**[Total 15 marks]**

### **Question 11**

A life office issues a 3-year unit-linked endowment policy to a man aged 40 under which level annual premiums of £1,000 are paid. £75% of the first premium and 102% of each subsequent premium is invested in units. There is a bid/offer spread in unit values, the bid price being 96% of the offer price.

A fund management charge of 0.75% of the value of the policyholder's fund is deducted at the end of each policy year.

The death benefit, which is payable at the end of the year of death, is £2,5000 or the bid value of the units if greater. The maturity value is equal to the bid value of the units.

The office incurs expenses of £300 at the start of the first year, £75 at the start of the second year and £25 at the start of the third year.

The rate of mortality is assumed to be 0.01 at each age and withdrawals may be ignored.

The unit fund is assumed to grow at 6% per annum at the fund is assumed to earn interest at 4% per annum.

- (a) Assuming that the office holds unit reserves equal to the bid value of the units and zero sterling reserves, calculate the profit emerging in each policy year. **[12 marks]**
- (b) Calculate the revised profit emerging each year assuming that the office sets up sterling reserves to ensure that the profit emerging in the second policy year is non-negative. Calculate also the required sterling reserves.

**[5 marks]**  
**[Total 17 marks]**

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**END OF EXAMINATION PAPER**