NATIONAL UNIVERSITY OGF SCIENCE AND TECHNOLOGY

INSURANCE AND ACTUARIAL SCIENCE DEPARTMENT

ACTUARIAL STATISTICS FOR INSURANCE II (CIN: 4011)

APRIL/MAY 2003 EXAMINATION   TIME ALLOWED : 3 HRS

INSTRUCTIONS TO CANDIDATES

1) Write your full name and student number on the answer booklet
2) Begin each question on a separate sheet.
3) Marks to each question are shown in brackets
4) Attempt all 10 questions

ADDITIONAL MATERIAL

1) An electric calculator
2) A copy of the Actuarial Examination Tables

QUESTION I

Losses from a certain type of risk follow a lognormal distribution with parameters μ = 5 and ⑨^2 = 1.44. Find the expected amount that the insurer of this risk will pay per original claim if reinsurance is effected using:

a) a proportional reinsurance arrangement with proportion 0.8.

[2 marks]

b) an individual excess of loss treaty with retention limit £1 000.

[4 marks]

[Total 6 marks]

Question 2

An actuarial student is planning to attend a conference in London and he must send in his room reservation immediately. The conference is so popular that some events will be held in Hotel A and others in Hotel B. The student does not yet know whether the particular event he wishes to attend will be held in Hotel A or B. The student is only planning to stay one night which will cost £66 at Hotel A and £62.40 at Hotel B. If the student stays in the wrong hotel there will be an extra taxi fare of £6.

a) If the student believes that the odds are 3 to 1 that the event he wants to attend will be at Hotel A where should the reservation be made to minimise the expected cost? **[3 marks]**

b) If the student believes that the odds are 5 to 1 that the event he wants to attend will be at Hotel A where should be reservation be made to minimise the expected cost? **[2 marks]**

[Total 5 marks]

<u>Question 3</u>

Consider a generalised linear model (GML) with independent Poisson responses

Y ~ Poisson, ③ where ③ = E [**Y**]

and structure g(③) = ☙

Comprising a linear predictor, ☙, linked to the mean response, ③, through a link function, g.

(i) Write down the log-likelihood for this model, based on observations {℘$_i$: i = 1, 2, …, t}, and show that this defines an exponential family. **[4 marks]**

(ii) Identify the canonical link function. **[2 marks]**

[Total 6 marks]

<u>Question 4</u>

The following table shows cumulative claims (in £ '000) for a motor insurance portfolio, for 3 accident years. Over this period, claims inflation may be assumed to be at the rate of 4% per annum. Use the inflation-adjusted chain ladder technique to derive a claims reserve in 2001 prices.

Accident Year	Development Year 0	1	2
1999	765	943	1010
2000	941	1011	
2001	921		

[Total 8 marks]

Question 5

A company has decided to introduce a No Claims Discount System for a certain class of business. This system will have 5 levels of discount: 0%, 15%, 30%, 50% and 75%. The rules for moving between these levels of discount will be:

a) following a year with one or more claims, a policyholder will move down one level of discount, or remain at the 0% discount level.

b) following a claim free year, a policyholder will move up one level of discount, or remain at the 75% discount level.

The company believes that the probability of a policyholder having a claim free year is 0.85. What proportion of policyholders at each level of discount should the company expect once the proportions have become stable?

[Total 8 marks]

Question 6

An insurance company is monitoring the length of time staff take to pick up telephones after they first ring. It is assumed that time follows an exponential distribution with parameter λ.

10 calls are monitored at random and the average response time is calculated as 3.672 seconds.

(i) a) Show that the Gamma distribution is the conjugate prior distribution for λ.

b) Assuming that the prior distribution for λ has mean 0.315 and standard deviation of 0.251, derive the posterior distribution of θ and calculate the Bayesian estimator of θ under quadratic loss. **[7 marks]**

(ii) A further 70 calls are monitored and have the same average response time of 3.672 seconds. Calculate the Bayesian estimator of θ under quadratic loss using all the data collected. **[2 marks]**

(iii) Comment on your answers in (i) and (ii) [**2 marks]**

[Total 11 marks]

<u>Question 7</u>

(i) Claims arrive as a Poisson process with rate ②. The claim sizes are independent, identically distributed random variables X_1, X_2, ----- with

$$P(X_i = k) = P_k, \qquad k = 1, \ldots, M, \ \sum_{K=1}^{M} P_k = 1$$

If premium loading factor is θ, show that the adjustment coefficient R satisfies :

$$\tfrac{1}{M} \log (1 + \theta) < R < \frac{2\theta m_1}{m_2},$$

where m_i = $[E_1^i]$, i = 1,2. **[7 marks]**

[The inequality $e^{Rx} \le \frac{\chi}{M} e^{RM} + 1 - \frac{x}{m} (0 \le x \le m)$ may be used without proof.]

(ii) If θ = 0.2and X_i is equally likely to be 1 or 2, determine an upper and a lower bound for **R,** and hence derive an upper bound on the probability of ruin when the initial surplus is ③.

[3 marks]

[Total 10 marks]

Question 8

An insurance company has a group life insurance policy which covers 2 classes of employees. The sum insured payable in the event of death is twice the annual salary of the employee. Within each class of employee, annual salaries are assumed to be normally distributed, and the relevant data are given in the following table:

Class of Employee	Number of Employees	Probability of Death	Mean Salary	Standard Deviation of Salaries
1	1 000	0.0001	10 000	2 000
2	500	0.0005	20 000	3 000

(i) Derive the moment generating function of aggregate claims, stating any assumptions made. **[6 marks]**

(ii) Calculate the mean and variance of aggregate claims. **[4 marks]**

[Total 10 marks]

Question 9

(i) Show that

$$\int_a^b \frac{1}{\sqrt{2\pi\sigma^2}}\ e^{-\frac{1}{2}\frac{(\ln x-\mu)^2}{\sigma^2}}\ dx = e^{\mu+\frac{1}{2}\sigma^2}\left[\Phi\left(\frac{\ln b-\mu-\sigma^2}{\sigma}\right)-\Phi\left(\frac{\ln a-\mu-\sigma^2}{\sigma}\right)\right]$$

[4 marks]

(ii) Individual claim amounts on a certain type of general insurance policy have a log-normal distribution, with mean 264 and standard deviation 346. A policyholder excess of 100 is a standard condition on each policy, so that the insurance company only covers the loss amount in excess of 100.

a) Calculate the expected claim size payable by the insurance company.

b) Next year, claims are expected to increase by 10%. Also, a new condition will be introduced on all policies so that the maximum amount that the insurance company will pay on any claim will be 1000. The policyholder excess will remain unchanged at 100.

Calculate the expected claim size payable by the insurance company. **[14 marks]**

[Total 18 marks]

Question 10

(i) State carefully the assumptions made for a single risk by Model 1 and Model 2 Empirical Bayes Credibility. **[5 marks]**

(ii) The table below shows the aggregate claims, denoted Y_{ij}, for each of three risks over five years, together with some summary statistics.

	Year	**1**	**2**	**3**	**4**	**5**	$\overline{Y_i}$	$\sum_{j=1}^{5}(Y_{ij}-\overline{Y_i})^2$
	1	1947	2594	1791	2340	2591	2253	545 193
Risk	2	1970	1497	1791	2034	2217	1902	297 619
	3	2191	2514	2217	2422	2821	2433	262 446

(a) Use Model 1 to calculate the credibility premium for the coming year for each of these three risks.

(b) Now suppose you have available a risk volume, denoted P_{ij}, corresponding the statistic. You are given the following values:

Risk	$\sum_{j=1}^{5} P_{ij}$
1	890
2	569
3	1 003

Using Model 2, the credibility premium per unit of risk volume for the coming year for risk number 1 is 12.92. Calculate the credibility premium per unit of risk volume for the coming year for risk number 2 and risk number 3. **[13 marks]**

[Total 18 marks]

END OF EXAMINATION!