NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

B. COMM (HONOURS) ACTUARIAL SCIENCE

ACTUARIAL MATHEMATICS IIA : CIN 4110

NOVEMBER/DECEMBER 2004 FIRST SEMESTER EXAMINATIONS

DURATION : 3 HOURS

INSTRUCTIONS TO CANDIDATES
1.   In addition to this paper you should have available Actuarial Tables and your own electronic calculator.
2.   Mark allocations are shown in brackets.
3.   Attempt all 14 questions.

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Question 1

(i)   Explain what is meant by the following:

$_{3|4}q_{[40]+1}$

(ii)   Calculate the value using A1967-70 mortality.

[3 marks]

Question 2

Show that the premium conversion relationship $A_u = 1 - d\ddot{a}_u$ holds for the joint life status xy. **[3 marks]**

Question 3

Define the following terms:

   Select mortality
   Ultimate mortality
   Select period **[3 marks]**

Question 4

A husband aged 50 and a wife aged 53 require an income of $10 000.00 per annum whilst both are alive, which reduces to $7 000.00 per annum payable to the survivor after one of the lives has died. The income is payable in arrears.

Calculate the single premium payable to cover this benefit.

Basis : Mortality   males: A1967-70 ultimate

                               Females: A1967-70 ultimate, reduced by 3 years

          Interest: 4% per annum

          Expenses: Nil

[4 marks]

Question 5

Derive a simplified expression, in the form of assurance functions payable on the first death, (and not involving integrals) for the expected present value of the benefits for a contingent assurance policy where S is paid immediately on the death of a life aged x provided she dies within the 5 year period following the death of a second life aged y.

[5 marks]

Question 6

Define the following functions in words, and give an expression for each of them in terms of an integral.

(i) $_{\infty}q_{\overset{1}{xy}}$

(ii) $\overline{A}_{\overset{2}{xy}}$

(iii) $\overline{A}^{1}_{x:y:n}$ **[6 marks]**

Question 7

Your office has been asked to quote a single premium for a contingent assurance policy providing $300 000.00 immediately on the death of a woman now aged 80 within 15 years, provided that at the date of her death a man now aged 60 has died. Your office uses the following basis:

         Mortality :   Males a(55) ultimate

                              Females a(55) ultimate

         Interest: 8% per annum

         Expenses: 10% of the single premium

(i)   Assuming that the two lives are independent, write down a formula for the single premium in terms of an integral.   **[2 marks]**

(ii)   State a suitable rule of approximate integration for evaluating this integral.

                                                             [1 mark]

(iii)   Would you subject the male life to stringent underwriting procedures? Give brief reasons for your answer.   **[3 marks]**

                                                           [Total : 6 marks]

Question 8

(i)   State the conditions necessary for gross premium retrospective and prospective reserves to be equal.   **[3 marks]**

(ii)   Demonstrate the equality of gross premium retrospective and prospective reserves for a whole life policy, given the conditions necessary for equality. **[4 marks]**
                                                                        [Total : 7 marks]

Question 9

In the context of a life office establishing reserves for its in force business, list four distinct ways of reducing policy values to allow for the incidence of expenses. In each case, state whether it applies to unit-linked or non-linked life assurance contracts (or to both), and state its advantages and disadvantages.   **[8 marks]**

Question 10

Give simplified expressions for single premiums for the following whole life contingent assurances for a sum assured of $20 000.00.

(a)   Payable on the death of (x) provided this occurs before the death of (y) or within n years after the death of (y).   **[4 marks]**

(b)   Payable on the death of (x) provided this occurs at least n years after the death of (y).   **[4 marks]**
                                                                        [Total : 8 marks]

Question 11

A life aged 40 purchases a deferred annuity of $2 000.00 per annum by level annual premiums during the period of deferment. The first payment of annuity is due on the life's 60th birthday, and payments are due annually thereafter.

The annuity is guaranteed for 5 years and continues for life thereafter. If the life dies during the deferred period, the premiums paid are returned to the life's estate, without interest at the end of the year of death.

Calculate the premium on the following basis:

Mortality	-	A1967-70 ultimate during the deferred period; a(55) males from age 60
Interest	-	4% per annum
Initial Expenses	-	40% of the first premium
Renewal expenses	-	5% of any premium after the first
Annuity expenses	-	$10 on each annuity payment. **[8 marks]**

Question 12

A life office issues a proposer aged 35 a whole life policy participating in profits for a sum assured of $10 000.00. The sum assured and reversionary bonuses are payable immediately on death.

Calculate the monthly premium payable for a maximum of 30 years. If the office assumes that future reversionary bonuses will be at a rate of 2.913% of the sum assured, compounded annually and vesting at the start of each policy year.

Basis: Mortality: A1967-70 select
      Interest: 6% per annum
      Initial expenses: $50.00 plus 2% of the basic sum assured
      Renewal expenses: $10 on each annual anniversary of the date of issue of the policy throughout the life of the policyholder.

                  $2^1/_2$% of each premium, after the first monthly premium.

[10 marks]

Question 13

100 people aged exactly 50 are each sold a 15-year endowment assurance policy with sum assured $100 000.00. The premiums are paid annually in advance, and the sum assured is paid on maturity or at the end of the year of death if earlier.

The life insurance company's assumptions are:

Mortality : A1967-70 ultimate, and the lives are independent with respect to mortality.
Interest : 6% per annum
Expenses : Initial : $300
        Renewal : 2.5% of each premium, including the first

Let P be the gross premium.

(i) State the gross future loss random variable for one policy at outset.
                          [3 marks]

(ii) Using your answer to (i) or otherwise, evaluate, in terms of P,
   (a)The mean and variance of the loss (in present value terms) for a single policy at outset.
   (b)The mean and variance of the loss (in present value terms) for the entire portfolio at outset.   **[7 marks]**

  Note : $A_{50:15}$. at 12.36% per annum = 0.20426

(iii) Show what values the gross annual premium P can take if the company requires that the probability of it incurs a loss (in present value terms) on the entire portfolio has to be less than 2.5 %. Use the normal approximation. **[4 marks]**

[Total : 14 marks]

Question 14

A life office issues a 4-year unit linked endowment assurance policy to a man aged 61 exact under which level premiums of £3 000 per annum are payable in advance. In the first year 98% of the premium is invested in units at the offer price, and in subsequent years the allocation percentage is 102%. In the first two years, the premium is used to buy capital units, In years three and four, premiums buy accumulation units. The bid/offer spread in the prices of both kinds of units is 4% of the offer price.

Annual management charges are:

Capital units	4.00%
Accumulation units	0.75% per annum

Management charges are deducted from the unit fund before death benefits are paid. If the policyholder dies during the term of the policy, then a death benefit equal to the greater of £7 000 or the bid value of the units is paid at the end of the year of death.

The office holds no sterling reserves. Assumptions are:

Rate of interest of unit fund:	12% per annum
Rate of interest of sterling fund	10% per annum
Risk discount rate	16% per annum
Mortality rate	0.02 per year
Surrender rate:	Nil
Initial expense	£90 plus 25% of first year premium
Renewal premium	£12 on each premium date except the first.

Assume that the office holds unit reserves equal to the full bid value of both types of units.

(a) Calculate the profit signature of the contract.
(b) Calculate the net present vale of the profit on the policy at its outset.

[Total : 15 marks]

END OF EXAMINATION PAPER!!!