

NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

B.COMM (HONOURS) DEGREE IN ACTUARIAL SCIENCE

ACTUARIAL MATHEMATICS IIA – CIN 4110

SUPPLEMENTARY EXAMINATION

JULY 2005 EXAMINATION

DURATION: 3 HOURS

Instructions to Candidates

1. Write your student number on the answer booklet
2. Begin each question on a separate sheet.
3. Marks to each question are shown in brackets
4. Attempt all 17 questions

ADDITIONAL MATERIAL

An electronic calculator

A copy of the Actuarial Examination Tables

1. In the context of a pension scheme, explain the term “prospective service benefit” and state one example. **[3 marks]**

2. In a select mortality investigation, $\theta_{x,r}$ corresponds to the number of deaths aged x next birthday at entry with duration r at the anniversary following death. $\theta_{x,r}$ divided by the appropriate central exposed to risk gives an estimate of $\mu_{[y]+t}$.

Derive the values of y and t to which this estimate applies. State clearly any assumptions used. **[3marks]**

3. A 25 year annual premium endowment assurance policy was sold to a life aged 40 exact at outset. Death benefits are payable at the end of the year of death. Calculate the Zillmerised net premium reserve at the end of the tenth year per unit sum assured.

Basis: Mortality: A1967-70 Select
Interest: 3% per annum
Initial expense: 2.5% of the sum assured.

[3 marks]

4. Two lives, each aged x , are subject to the same mortality table. According to the mortality table and a certain rate of interest, $A_x = 0.5$ and $A_{xx} = 0.8$.

Calculate $A_{\overline{xx}}$, using the same mortality table and interest rate. **[3 marks]**

5. (i) A life insurance policy provides a benefit of £10,000 payable immediately on the death of a life (x) , if (x) dies after a life (y) . Express in integral form the expected present value of the benefit under this policy. **[2 marks]**

(ii) Set out, giving a reason, the most appropriate annuity factor to value annual premiums payable under the policy. **[1 mark]**

[Total: 3 marks]

6. A healthy life aged exactly 35 has a policy providing an income benefit of £50 per week payable during sickness. The benefit is not payable beyond age 60. There is no deferred or waiting period.

Calculate the present value of this benefit.

Basis: Mortality: English Life Table No. 12-Males

Sickness: Manchester Unity Sickness Experience 1893/97

Occupation Group AHJ

Interest: 4% per annum.

[4 marks]

7. A life insurance company sells an annual premium whole life assurance policy with benefits payable at the end of the year of death. Expenses are incurred at the start of each year, and claim expenses are nil.

(a) Write down a recursive relationship between the gross premium reserves at successive durations, calculated on the premium basis. Define all symbols used.

(b) Explain the meaning of this formula. **[5 marks]**

8. Calculate $A_{30:30:30}^1$ using A1967-70 mortality and interest of 4% per annum.

[5 marks]

9. The random variables T_x and T_y represent the exact future lifetimes of two lives aged x and y respectively.

Let the random variable $g(T)$ take the following values:

$$g(T) = \begin{cases} a_n & \text{if } \max \{T_x, T_y\} \leq n \\ a_{\max\{T_x, T_y\}} & \text{if } \max \{T_x, T_y\} > n \end{cases}$$

(i) Describe the benefit which has present value equal to $g(T)$. **[3 marks]**

(ii) Express $E[g(T)]$ as concisely as possible in the form of an annuity function. **[2 marks]**
[Total: 5 marks]

10. Define the term “asset share” in the context of a with-profit policy. **[5 marks]**

11. An annuity of 1 is payable annually in arrears while at least one of two lives, (x) and (y) , is alive.

Derive an expression in terms of joint – life and single life functions for the variance of the present value of the annuity. **[5 marks]**

12. In the context of a life insurance contract, explain how an asset share may be built up using a recursive formula. **[5 marks]**

13. (i) On 1 January 1990 a life insurance company issued a 20-year annual premium without profits endowment assurance policy to a life then aged exactly 40, which is still in force. The sum assured of £100,000 is payable at the end of the year of death within the term of the policy, or on survival. The company values the policy using a modified net premium method, with a Zillmer adjustment.

Calculate the reserve for the policy on 31 December 1999.

Basis: Mortality: A1967 – 70 Select
 Interest: 4% per annum
 Zillmer adjustment: 2% of the sum assured. **[3 marks]**

(ii) Without carrying out any further calculations, explain how the value of the policy would differ if the company used a Zillmer adjustment of 1% of the sum assured, with the same mortality and interest assumptions. **[2 marks]**
[Total: 5 marks]

14. A life insurance company issues a special reversionary annuity contract. Under the contract an annuity of £10,000 per annum is payable monthly for life, to a female life now aged exactly 60, on the death of a male life now aged exactly 65,

provided the male life dies within 10 years of the start date of the policy. Payments commence on the first monthly policy anniversary after the date of death.

Calculate the single premium required for the contract.-

Basis: Mortality: a(55) Ultimate mortality, male or female as appropriate.
Interest: 6% per annum
Expenses: none **[5 marks]**

15. Describe three types of bonus that may be given to a with profits contract. **[6 marks]**

16. A life insurance company issues a 4 year unit-linked policy with a level premium of £1,000 payable annually in advance to a life aged exactly 61. The death benefit at the end of the year of death is £4,000, or the bid value of the units if greater. The maturity value is the bid value of the units.

95% of each premium is invested in units at the offer price. The bid price is 95% of the offer price. Premiums payable in the first two years are invested in capital units which are subject to a management charge of 6% per annum. Subsequent premiums are invested in accumulation units for which the management charge is 1% per annum. Management charges are deducted at the end of each year from the bid value of units before benefits are paid.

Capital units are units actuarially funded using factors of $A_{61+t:4-t}$ calculated using A1967 – 70 Ultimate with 5% per annum interest for $t = 0, 1, 2$ and 3.

The company uses the following assumptions to profit test this contract:

Rate of interest on unit investments:	8% per annum
Rate of interest on sterling fund:	4% per annum
Mortality:	A1967 – 70 Ultimate
Initial Expenses:	£100 plus 20% of the first premium
Renewal expenses:	£20 on the first policy anniversary, and increasing with inflation at 5% per annum on each subsequent anniversary.

- (i) Using a risk discount rate of 12% per annum calculate the expected net present value of the profit on this contract. **[12 marks]**
- (ii) Without performing any further calculations, state with reasons whether your answer in (i) would be higher or lower for each of the following, if

- (a) the risk discount rate were 10% per annum
 - (b) the policyholder were aged 50 exactly
 - (c) capital units were actuarially funded at 4% per annum. **[5 marks]**
- [Total: 17 marks]**

17. A man aged exactly 30 effected a 35 year with profit endowment assurance for a sum assured of £50,000. Level annual premiums are payable throughout the policy term, ceasing on earlier death. The sum assured, with attaching bonuses, is payable at the end of the year of death, or on maturity. Compound reversionary bonuses vest at the end of each policy year.

- (i) Show that the premium (to the nearest £1) is £990 per annum using the following basis.

Mortality: A1967 – 70 Ultimate
 Interest: 6% per annum
 Expenses: Initial: £ 250 PLUS 60% of the annual premium
 Renewal: 2.5% of second and subsequent premiums
 Bonuses: 1.923% per annum **[7 marks]**

- (ii) The random variables T_x and K_x represent the exact future lifetime and the curtate future lifetime of a life aged x , respectively. Using T_x , K_x or both, express, in stochastic form, the gross future loss random variable for this policy at duration t , where t is an integer and $0 < t < 35$. Use those elements of the basis set out in part (i) as needed. Assume bonus declarations have been in line with the original bonus loadings.

[3 marks]

- (iii) Immediately before the 11th premium is due, and just after the 10th bonus has brought the sum assured plus accumulated bonuses to £60,000, the policy holder wishes to convert the policy to a non-profit whole life policy, with premiums of an unchanged amount payable until death.

Using the mortality and interest elements of the premium basis set out in part (i), and allowing for renewal expenses of 2.5% of all future premiums as well as an alteration expense of £100, calculate the revised sum assured.

[6 marks]

- (iv) State one other consideration, if any, that the office should take into account before completing the alteration in (iii), and explain why they should do so.

[2 marks]

[Total: 18 marks]

END OF EXAMINATION!
