NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

B.COMM (ACTUARIAL SCIENCE) HONOURS DEGREE

ACTUARIAL MATHEMATICS IIA – CIN 4110

JULY/AUGUST 2006 SUPPLEMENTARY EXAMINATION

DURATION: 3 HOURS

Instructions To Candidates

1. ***Write your student number on the answer booklet.***
2. ***Begin each question on a separate sheet.***
3. ***Marks to each question are shown in brackets***
4. ***Attempt all 9 questions.***

Additional Material

1. ***An electronic calculator.***
2. ***Two copies of Actuarial Examinations Tables.***

Question One

Show that if mortality can be represented by an addition of a constant k to the standard force of mortality, then annuity functions can be calculated using the standard mortality tables with an adjustment to the force of interest. **[Total: 4 marks]**

Question Two

a. Define the following terms in the context of an application for a temporary life assurance contract, and give an example for each:

i. Temporary initial selection
ii. Self-selection **[4 marks]**

b. List the topics that you would expect to be included in the questions on a proposal form which a life company might use for risk classification purposes. The contracts concerned are five-year temporary assurance policies and the expected applicants are men aged 25-45. **[6 marks]**

[Total : 10 marks]

Question Three

Write the net future loss random variable at policy duration t for a with profit whole life with compound bonuses. Assume that premiums are paid continuously, bonuses are added at the end of year and benefits are paid immediately on death. **[Total: 6 marks]**

Question Four

Defining xy as the joint life status for lives aged x and y respectively prove that
${}_nq^2_{xy} = {}_nq_{xy}^{1} - {}_np_x\,{}_nq_y$. **[Total: 8 marks]**

Question Five

A life insurance company issues an annuity contract to a man aged 65 exact and his wife aged 62 exact. Under the contract, an annuity of $20,000 per annum is guaranteed payable for a period of 5 years and thereafter during the lifetime of the man. On the man's death, an annuity of $10,000 per annum is payable to his wife, if she is then alive. This annuity commences on the monthly payment date next following, or coincident with, the date of his death or from the 5th policy anniversary, if later and is payable for the lifetime of his wife. Annuities are payable monthly in advance.

Calculate the single premium required for the contract.
Basis: Mortality PMA92C20 for the male and PFA92C20 for the female
Interest 4% per annum
Expenses none **[Total: 10 marks]**

Question Six

On 1 January 2000, a life insurance company issued an endowment assurance policy to a life aged exactly 50 for a term of 10 years. Under the policy, a sum assured of $100,000 is payable on survival to age 60 exact or at the end of the year of death on earlier death. Level premiums are payable annually in advance for 10 years or until earlier death. On 1 January 2003, the policy is still in force and the life insurance company calculates on a prospective basis both the gross premium reserve and the net premium reserve for the policy at this date, using the assumptions shown below. The same assumptions were used to calculate the gross premium at inception as follows:

Mortality:	AM92 Ultimate
Interest:	4% per annum
Expenses:	Initial: $300 incurred at the outset
Renewal:	5% of each premium

(i) Calculate the gross premium reserve as at 1 January 2003. **[3 marks]**

(ii) Calculate the net premium reserve, with Zillmer adjustment, as at 1 January 2003. Identify clearly the Zillmer adjustment. **[4 marks]**

(iii) Explain why the net premium reserve with Zillmer adjustment calculated in part (ii) might be used in preference to the net premium reserve with no Zillmer adjustment, calculated as at 1 January 2003, using the same assumptions. **[2 marks]**

(iv) Assume instead that the life insurance company calculated the gross premium reserve as at 1 January 2003 using a rate of interest of 3.5% per annum following a general fall in market interest rates, with all other assumptions unchanged. Assume also that the net premium reserve with a Zillmer adjustment, calculated in part (ii), is unchanged. State, giving a reason, whether you consider it appropriate to use this unchanged net premium reserve with a Zillmer adjustment for reserving purposes. **[3 marks]**

[Total: 12 marks]

Question Seven

A life insurance company issued a with profits whole life policy to a life aged 20 exact, on 1 July 2002. Under the policy, the basic sum assured of $100,000 and attaching bonuses are payable immediately on death. The company declares simple reversionary bonuses at the start of each year. Level premiums are payable annually in advance under the policy.

(i) Give an expression for the gross future loss random variable under the policy at the outset. Define symbols where necessary. **[5 marks]**

(ii) Calculate the annual premium, using the equivalence principle.

Basis: Mortality AM92 Select
Interest 6% per annum
Bonus loading 3% simple per annum
Expenses Initial $200
Renewal 5% of each premium payable in the second and subsequent years
Assume bonus entitlement earned immediately on payment of premium. **[5 marks]**

(iii) On 30 June 2005 the policy is still in force. A total of $10,000 has been declared as a simple bonus to date on the policy. The company calculates provisions for the policy using a gross premium prospective basis, with the following assumptions:
Mortality AM92 Ultimate Interest 4%
Bonus loading 4% per annum simple
Renewal expenses 5% of each premium
Calculate the provision for the policy as at 30 June 2005. **[5 marks]**

[Total: 15 marks]

Question Eight

You are a consulting actuary to a client who wishes to invest $1million now to provide an immediate income for his partner and himself in retirement. Both the client and his partner are aged 60 exact. The client wishes to provide a payment annually in advance each year while either he or his partner is alive. He wishes the amount of the payment to be $\$I^* (1.05)^t$, $t = 0,1, 2.....$

where I denotes the amount of the initial payment and t denotes the curtate duration in years since the inception of the policy.

The client further requests that he wishes the amount of the initial payment *I* to be such that the capital of $1million is at least 95% likely to be sufficient to provide the required payments and he asks you to advise what the maximum value of the initial payment *I* should be.

In carrying out the calculations, you assume that the only source of random variation is the future mortality of the client and his partner. Calculate the required value of *I* based on the following assumptions.

Mortality: The client and his partner are independent with respect to mortality and are each subject to the mortality of PMA92C20.
Rate of future investment returns: 6% p.a.
Expenses: none **[Total: 12 marks]**

Question Nine

A life insurance company issues a 4-year unit-linked endowment policy to a life aged 61 exact under which level premiums of $5,000 per annum are payable annually in advance.
102% of each premium is invested in units at the offer price. In the first year the premium is used to buy capital units. In subsequent years premiums are used to buy accumulation units. There is a bid/offer spread in the prices of both types of units of 6% of the offer price.

The annual management charges are as follows:

Capital units:	5% per annum
Accumulation units:	1% per annum

Management charges are deducted from the unit fund before death and surrender benefits are paid. If the policyholder dies during the term of the policy then a death benefit equal to the bid value of the units is paid at the end of the year of death. The policyholder may surrender the policy only at the end of each year. The surrender value is equal to the bid value of the accumulation units plus a proportion of the bid value of the capital units. The proportion depends on the year of surrender as follows:

Year of Surrender	*Proportion of capital units paid out*
1	0.8227
2	0.8638
3	0.9070
4	1.0000

The company uses the following assumptions in its profit test of this contract:

Rate of interest on unit investments:	10% per annum
Rate of interest on sterling fund:	8% per annum
Mortality:	A1967–70 Ultimate
Initial expenses:	$150 plus 20% of the first premium

Surrender rates: 10% of all policies still in force at the end of each of the first, second and third years
Renewal expenses: $20 on each premium payment date except the first plus 0.5% of each premium except the first.

(i) Using a risk discount rate of 12% per annum, calculate the expected net present value of profit on this contract:

(a) Assuming that the company holds no sterling reserves and holds unit reserves equal to the full bid value of both types of unit **[10 marks]**

(b) Assuming that the company sets up negative sterling reserves, which zeroise all expected, net profits in all years except the first. **[4 marks]**

(c) Assuming that the company holds a proportion $A_{61+t,\ 4-t}^{\sim}$, (calculated at 4%) of the full number of capital units just after the payment of the premium due at time t (t = 0, 1, 2 and 3). **[7 marks]**

(ii) For this type of contract, state with reasons which of (i)(b) and (i)(c) you regard as the more appropriate way of bringing forward profit. **[2 marks]**

[Total: 23 marks]

-END OF EXAMINATION-