

NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF COMMERCE

DEPARTMENT OF INSURANCE AND ACTUARIAL SCIENCE

B.COMM (HONOURS) DEGREE IN ACTUARIAL SCIENCE

ACTUARIAL MATHEMATICS 11A (CIN 4110)

APRIL / MAY 2009-FIRST SEMESTER EXAMINATIONS

TIME ALLOWED: 3 HOURS

INSTRUCTIONS TO CANDIDATES

- 1 Answer all 12 questions.
 - 2 Write clearly and begin each question on a new page.
 - 3 In addition to this paper a candidate should have a **copy of 2002 actuarial tables** and a non-programmable scientific calculator.
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- Q1** A 20-year temporary annuity-due of 1 per annum is issued to a life aged 50 exact.
- (a) Express the expected present value of the annuity in terms of an assurance function.
 - (b) Hence calculate the value using the mortality table AM92 Ultimate with 4% interest. [3 marks]
- Q2** A life aged 50 who is subject to mortality of AM92 Select Table, effects a pure endowment policy with a term of 20 years for a sum assured of \$ 10 000.
- (i) Write down the present value of the benefit under this contract, regarded as a random variable. [2 marks]
 - (ii) Assuming an effective rate of interest of 4% per annum, calculate the Mean and the Variance of the present value of the benefit available under the contract. [3 marks]
- [Total 5 marks]
- Q3** An office issues a block of 400 without profits endowment assurances, each for a term of 25 years to lives aged exactly 35. The sum assured under each policy is \$ 10 000 and the premium is \$ 260 per annum, payable continuously during the term. The sum assured is payable immediately on death, if death occurs within the term of the policy. Assuming that the office will earn 4% interest per annum, that the future lifetime of the lives may be described statistically in terms

of the AM92 Ultimate table, and that expenses may be ignored, Find the Mean present value of the profit to the office on the block of policies. [5 marks]

Q4 A life insurance company issued a with profits whole life policy to a life aged 20 exact, on 1 July 2002. Under the policy, the basic sum assured of \$ 100 000 and attaching bonuses are payable immediately on death. The company declares simple reversionary bonuses at the start of each year. Level premiums are payable annually in advance under the policy.

(i) Give an expression for the gross future loss random variable under the policy at the outset. Define symbols where necessary. [3]

(ii) Calculate the annual premium, using the equivalence principle.

Basis:

Mortality		AM92 Select
Interest		6% per annum
Bonus loading		3% simple per annum
Expenses	Initial	\$ 200
	Renewal	5% of each premium payable in the second and subsequent years

Assume bonus entitlement earned immediately on payment of premium. [4 marks]

(iii) On 30 June 2005 the policy is still in force. A total of \$ 10 000 has been declared as a simple bonus to date on the policy.

The company calculates for the policy using a gross premium prospective basis, with the following assumptions:

Mortality	AM92 Ultimate
Interest	4% per annum
Bonus loading	4% per annum simple
Renewals expenses	5% of each premium

Calculate the provision for the policy as at 30 June 2005. [4]
[Total 11 marks]

Q5 A whole life assurance policy was issued to a life aged 45, the sum assured of \$ 50 000 is payable at the end of year of death. Level monthly premiums are payable for a maximum of 20 years.

Expenses:

- Initial \$ 150
60% of the first year's premium arising when the premiums are paid.
- Renewal 3% of all premiums after the 1st year (when they are paid)
- Per Policy expenses - at the start of each year when a premium is paid = \$ 50 at time 0 (at the start of 1st year and increasing by 2,9126% per annum compound)
- Claim expenses- \$ 100 at current rate but increasing by 2,9126% per annum compound (that is if claim is paid at time t, t = 1, 2, 3,, expenses is $100(1.02916)^t$.

Basis: AM92 Ultimate Table at 6% per annum.

Calculate the monthly premium. [16 marks]

Q6 (i) Write down in the form of symbols, and also explain in words, the expressions "death strain at risk", "expected death strain" and "actual death strain". [6 marks]

- (ii) A life insurance company issues the following policies:
- 15-year term assurance with a sum assured of \$ 150 000 where the death benefit is payable at the end of the year of death.
 - 15-pure endowment assurances with a sum assured of \$75000.
 - 5-year single premium temporary immediate annuities with an annual benefit payable in arrear of \$25000.

On 1 January 2002, the company sold 5000 term assurance policies and 2000 pure endowment policies to male lives aged 45 exact and 1000 temporary immediate annuity policies to male lives aged 55 exact. For the term assurance and pure endowment policies, premiums are payable annually in advance. During the first two years, there were fifteen actual deaths from the term assurance policies written and five actual deaths from each of the other two types of policy written.

- (a) Calculate the death strain at risk for each type of policy during 2004.
- (b) During 2004, there were eight actual deaths from the term assurance policies written and one actual death from each of the other two types of policy written. Calculate the actual mortality profit or loss to the office in the year 2004.

Basis:

Interest 4% per annum

Mortality AM92 Ultimate for term assurances and pure endowments, and PMA92C20 for annuities.

[13 marks]

[Total 19 marks]

- Q7** In a certain country, pension funds always provide pensions to retiring employees. At the point of retirement, the fund can choose to buy an annuity from a life insurance company, or pay the pension directly themselves on an ongoing basis.

A mortality study of pensioners has established that the experience of those whose pension is received through annuities paid by insurance companies is lighter than the experience of those being paid directly by pension funds.

Explain why the mortality experiences of the two groups differ.

[4 marks]

- Q8** A pure endowment policy for a term of n - years payable by a single premium is issued to lives aged x at entry.

(i) Derive Thiele's differential equation for ${}_t\bar{V}$, the reserve for this policy at time t ($0 < t < n$). [4 marks]

(ii) Explain the effect of each term in your answer to (i). [2marks]

[Total 6 marks]

- Q9** A life insurance company issues identical deferred annuities to each of 100 women aged 63 exact. The benefit is \$ 5 000 per annum payable continuously from a woman's 65th birthday, if still alive at that time, and for life thereafter.

(i) Write down an expression for the random variable for the present value of future benefits for one policy at outset. [3]

(ii) Calculate the total expected present value at outset of these annuities.

Basis: Mortality: PFA92C20
Interest: 4% per annum [2 marks]

(iii) Calculate the variance of the present value at the outset of these annuities, using the same basis as in part (ii). [8 marks]

[Total 13 marks]

Q10 (i) In the context of net premiums and reserves, state the conditions necessary for equality of prospective and retrospective reserves. [2 marks]

(ii) Give two reasons why, in practice, these conditions may not hold. [2 marks]

[Total 4 marks]

Q11 A life office issued 750 identical 25-year temporary assurance policies to lives aged 30 exact each with a sum assured of \$75 000 payable at the end of year of death. Premiums are payable annually in advance for 20 years or until earlier death.

(i) Show that the annual net premium for each policy is approximately equal to \$104 using the basis given below. [2]

(ii) Calculate the net premium reserve per policy at the start and at the end of the 20th year of the policy. [4 marks]

(iii) Calculate the mortality profit or loss to the life office during the 20th year if twelve policyholders die during the first nineteen years of the policies and two policyholders die during the 20th year. [4 marks]

Basis:
Mortality: AM92 Ultimate
Interest: 4% per annum.

[Total 10 marks]

Q12 On the basis of E.L.T No 12 Males, calculate the probability that a life aged 30 will;

(i) Survive to age 40. [2 marks]

(ii) Die between his 40th birthday and his 50th birthday. [2 marks]

[Total 4 marks]

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