NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF INSURANCE AND ACTUARIAL SCIENCE

B.COMM (HONS) DEGREE IN ACTUARIAL SCIENCE

AUGUST 2009 EXAMINATIONS

SUBJECT: ACTUARIAL MATHEMATICS II B (CIN 4110)

TIME ALLOWED: 3 HOURS

INSTRUCTIONS TO CANDIDATES

1. Answer all 13 questions.
2. Write clearly and begin each question on a new page.
3. In addition to this paper a candidate should have a **copy of 2002 edition actuarial tables** and a non-programmable scientific calculator.

Q1 Explain the difference between a profit signature and a profit vector. [2 marks]

Q2 A pension scheme provides benefit on death in service of 4 times the member's salary at the date of death. Normal pension age is 65. State a formula, without using commutation functions, for the present value of this benefit to a life aged 35 exact with salary of $ 25 000 who has just received a salary increase. Define all symbols used. [6 marks]

Q3 You are given the following statistics in relation to the mortality experience of Actuaria and its province Giro:

	ACTUARIA		GIRO	
Age	Exposed to risks	Number of deaths	Exposed to risks	Number of deaths
0-19	300 000	25	12 000	2
20-39	275 000	35	10 000	3
40-59	200 000	100	9 000	6
60-79	175 000	500	8 000	50

(i) Explain , giving formula, the term Standardised Mortality Ratio (SMR). Define all the symbols that you use. [3 marks]

(ii) Comment on the relative mortality of the province, by calculating the SMR for Giro. [5 marks]

Q4 Using the PMA92C20 table for both lives calculate:

(a) $\mu_{65:60}$

(b) ${}_5 p_{65:60}$

(c) ${}_2 q^{1}_{65:65}$ [6 marks]

Q5 A member of a pension scheme is aged 55 exact, and joined the scheme at age 35 exact. She earned a salary of $ 40 000 in the 12 months preceding valuation date.

The scheme provides a pension on retirement for any reason of 1/80 th of final pensionable salary for each year of service, with fractions counting proportionately. Final pensionable salary is defined as the average salary over the three years prior to retirement.

Using the function and symbols defined in, and assumptions underlying, the Example Pension Scheme Table in the Actuarial Tables:

(i) Calculate the expected present value now of this member's total pension. [5 marks]

(ii) Calculate the contribution rate required, as a percentage of salary, to fund the future service element of the pension. [3 marks]

Q6 A population is subject to two modes of decrement, α and β. In the single decrement tables:

$${}_tP_{60}^{\alpha} = \frac{40-t}{40} \qquad \text{for } 0 \le t \le 40$$

and

$${}_tP_{60}^{\beta} = \left(\frac{40-t}{40}\right)^2 \qquad \text{for } 0 \le t \le 40$$

Calculate the value of $(aq)_{60}^{\alpha}$. [7 marks]

Q7 Explain how an insurance company uses risk classification to control the profitability of its life insurance business. [6 marks]

Q8 Define $\ddot{a}^{(12)}_{60:50:\overline{20}|}$ fully in words and calculate its value using PMA92C20 and PFA92C20 tables for the two lives respectively at 4% interest. [6 marks]

Q9 A life insurance company issues a three year unit-linked endowment assurance contract to a male life aged 62 exact under which level annual premiums of $ 10 000 are payable in advance throughout the term of the policy or until earlier of death. 85% of each year's premium is invested in units at the offer price.

There is a bid-offer spread in each unit values, with the bid price being 95% of the offer price.

There is an annual management charge of 1,25% of the bid value of units. Management charges are deducted at the end of each year, before death or maturity benefits are paid.

On death of the policyholder during the term of the policy, there is a benefit payable at the end of the year of death of $ 20 000, or the bid value of the units allocated to the policy, if greater. On maturity, 115% of the full bid value of the units is payable.

The company holds unit provisions equal to the full bid value of the units. It sets up non-unit provisions to zeroise any negative non-unit fund cashflows, other than those occurring in the first year.

The life insurance company uses the following assumptions in carrying out profit tests of this contract:

Mortality:		AM92 Ultimate
Expenses:	Initial	$ 600
	Renewal	$ 100 at the start of each of the second and third policy years
Unit fund growth rate:		8% per annum
Non-unit fund interest rate:		4% per annum

Non-unit fund provision basis: AM92 Ultimate mortality, interest 4% p.a

Risk discount rate: 15% per annum.

Calculate the profit margin on the contract. [16 marks]

Q10 (i) Define the following terms without giving detailed formulae:

(a) Crude mortality rate
(b) Directly Standardized Mortality Rate
(c) Indirectly Standardized Mortality Rate

[3 marks]

(ii) The data in the following table are taken from data published by the Office of National Statistics in 2001.

	England and Wales		**Tyne and Wear**	
	Population	Number of Births	Population	Number of births
Under 25	3 149 000	153 000	71 000	4 000
25-35	3 769 000	339 000	74 000	6 000
35+	3 927 000	103 000	82 000	1 000

(a) Using the population for England and Wales as the standard population, calculate crude birth rates and the directly and indirectly standardized birth rates for Tyne and Wear.

(b) State an advantage of using the Indirectly Standardised Birth Rate and comment briefly on the answers you have obtained. [10 marks]

[Total: 13 marks]

Q11 (i) Calculate the expected present value of an annuity-due of 1 per annum payable annually in advance until the death of the last survivor of two lives using the following basis:

First life: male aged 70, mortality table PMA92C20
Second life: female aged 67, mortality table PFA92C20
Rate of interest: 4% per annum [3 marks]

(ii) Give an expression for the Variance of the annuity-due in terms of annuity functions. [6 marks]

[Total: 9 marks]

Q12 A multiple decrement table is subject to two forces of decrement, α and β. Under the assumption of uniform distribution of the independent decrements over each year of age, $(aq)_x^{\alpha} = 0.15$ and $(aq)_x^{\beta} = 0.07$.

Calculate q_x^{α} and q_x^{β}. [5 marks]

Q13 Identify and explain any four forms of mortality selection. [8 marks]

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