NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

B COMM. (HONOURS) ACTUARIAL SCIENCE

ACTUARIAL STATISTICS III (STOCHASTIC MODELLING) : CIN 4111

NOVEMBER/DECEMBER 2004 FIRST SEMESTER EXAMINATION

DURATION : 3 HOURS

Instructions To Candidates

1. Attempt all questions, beginning your answer to each question on a separate sheet.
2. Mark allocations are shown in brackets

Additional Examination Material
Non-programmable electronic calculator
Tables of Actuarial Examinations (Actuarial Tables)

Quest6in 1

In the context of a discrete time Markov chain, define the terms:

(i) persistent **[2 marks]**
(ii) transient **[2 marks]**
(iii) aperiodic **[2 marks]**
[Total: 6 marks]

Question 2

A No Claims Discount (NCD) scheme for motor insurance has three levels of discount ; 0%, 20% and 60%. The rules for moving between these levels at the end of each year are as follows:

If the driver has made no claims in the previous year, she/he moves to the next higher level of discount, or remains at 60% discount. If the driver has made one or more claims in the year, she/he moves to the next lower level of discount, or remains at 0%.

The full premium for each year is $1 000. Let q denote the probability that a driver makes a claim in a year.

(i) Derive expressions in terms of q for the stationery distribution for this NCD Scheme.
[6 marks]

(ii) After a very long time in the scheme, a driver makes a claim. Calculate the expected value of the premium the driver will be asked to pay in the following year assuming:

(a) $q = 0.1$
(b) $q = 0.2$ **[6 marks]**

(iii) Comment on your answers to parts (ii) (a) and (ii) (b) above. **[4 marks]**
[Total: 16 marks]

Question 3

Let $X_1, X_2, X_3\ldots$ be identically and independently distributed with $P(X_1 = ½) = p$ and $P(X_1 = -½) = 1 – p$.

Let

$$F_n = \sigma \left\{X_1, X_2, \ldots\ldots X_n\right\}$$ and consider the process

$$Z_n = \pi_{i=1}^{n}(1 + x_i) \quad , Z_0 = 1$$

For what value(s) of p is Z_n?

(a) a supermartingale **[3 marks]**

(b) a submartingale **[3 marks]**

(c) a martingale **[3 marks]**

(Give reasons for your answers). **[Total: 9 marks]**

Question 4

Let X_t be an Ornstein – Uhlenbeck process satisfying

$dX_t = -a(X_t – b)\, dt + \sigma dB_t$ where $a > 0$ and B_t is a standard Brownian motion. Let $S(x)$ be the function

$$S(x) = \int_0^x \exp\left(\frac{a(y-b)^2}{\sigma^2}\right)dy$$

Show that $S(X_t)$ is a local martingale. **[7 marks]**

Question 5

A Markov chain has states labeled 0, 1 and 2 with transition matrix

$$P = \begin{pmatrix} ½ & 0 & ½ \\ 1 & 0 & 0 \\ ¼ & ½ & ¼ \end{pmatrix}$$

(i) Find P_{01} (2) **[3 marks]**

(ii) If the chain is in state 1 at time n = 5, what is the probability that it is in state 0 at time n = 7? **[3 marks]**

(iii) Does this chain have a stationary distribution? Is it unique? **[3 marks]**

(iv) Now suppose that $\underline{p}(0) = (^1/_3,\ ^1/_3,\ ^1/_3)$.

Find $P(X_0 = i \mid X_3 = 2)$ for $i = 0, 1, 2$

(i.e. find the initial distribution of the chain given that the chain is in state 2 after 3 steps.)

[5 marks]

[Total: 14 marks]

Question 6

Let X_1, X_2, ….X_3……be independent and identically distributed random variables with $E\left(|X_1|\right) < \infty$ and let $\mu = E(X_1)$

Let $S_0 = 0$ and $S_n = X_1 + X_2 + \ldots\ldots + X_n$ for $n \geq 1$.

Show that $M_n = S_n - \mu_n$ is a martingale relative to an appropriate filtration which you should specify.

[6 marks]

Question 7

The following AR(1) model

$Y_t - \mu = \phi(Y_{t-1} - \mu) + Z_t$ is fitted to a series of length 40. The fitted parameter values are:

$\hat{\Phi} = -0.7684$, $\hat{\mu} = 100.57$, $\hat{\sigma}_z^2 = 114.43$

The last 10 observations in the series are:

112, 101, 89, 116, 84, 118, 72, 118, 86, 96

(i) Forecast the next 3 observations in the series. **[6 marks]**

(ii) Find the 95% confidence intervals for each forecast in (i). **[9 marks]**

[Total : 15 marks]

Question 8

Let B be a Brownian motion (with $B_0 = 0$) and let $Y_t = B_t^n$ (i.e. nth power of B_t). By applying Ito's formula to the function $f(x) = x^n$, show that Y_t satisfies the following stochastic differential equation:

$$dY_t = nY_t^{1-\frac{1}{n}}\,dB_t + \frac{n(n-1)}{2}Y_t^{1-\frac{2}{n}}\,dt$$

[6 marks]

END OF EXAMINATION PAPER!!!