

**NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY**

**B. COMM (HONOURS) DEGREE IN ACTUARIAL SCIENCE**

**FINANCIAL ECONOMICS : CIN 4115**

**JANUARY 2004 1<sup>ST</sup> SEMESTER EXAMINATION**

**DURATION : 2 HOURS**

**INSTRUCTIONS TO CANDIDATES**

1. Attempt all questions
  2. You must not start writing your answers in the booklet until instructed to do so by the invigilator.
  3. In addition to this question paper, you require an electronic calculator (non-programmable)
- 

1. Investment project A has a rate of return that is uniformly distributed on [2%, 8%]. Project B has a rate of return that will either be 2%, 4%, 6% or 8%, each with probability  $\frac{1}{4}$ .

(a) Calculate the mean, variance and shortfall probability with level 4% for each project. **[6 marks]**

(b) An investor has the quadratic utility function:

$$U(x) = a + bx + cx^2$$

Under what circumstances will this investor choose investment project B. (You need to prove your result). **[4 marks]**

**[Total 10 marks]**

2. (a) In the context of “Modern Portfolio Analysis” briefly explain the concept of “stochastic dominance”. **[3 marks]**

(b) An investor can choose one or other of two investment portfolios A and B, which have independent rates of return. Both are exponentially distributed with parameter  $\lambda_A$  for the rate of return on A and  $\lambda_B$  for the rate of return on B, respectively. Assume  $\lambda_A < \lambda_B$ . (You may use that the exponential distribution with parameter  $\lambda$  has the density  $f_\lambda(x) = \lambda e^{-\lambda x}$  for  $x \geq 0$ )

State and prove the first order Stochastic Dominance Theorem for portfolios with exponentially distributed rates of return. **[9 marks]**

3. You are given that a portfolio consists of two assets A and B. These two assets give the following returns (Note: The investment portfolios are not independent).

<b>Probability</b>	<b>A</b>	<b>B</b>
$\frac{1}{3}$	30	45
$\frac{1}{3}$	75	60
$\frac{1}{3}$	90	75

- (i) Compute the expected return and variance for each asset [2 marks]
  - (ii) If short sales are allowed find the portfolio, p with the smallest variance. Find the mean and variance of this portfolio. [8 marks]
  - (iii) If short sales are disallowed comment on your answer(s) to part (ii) [2 marks]
- [Total 12 marks]**

4. (a) Explain the differences between the Capital Market Line and the Security Market Line. [3 marks]

(b) (i) State the Arbitrage Pricing Theorem [4 marks]

(ii) How can the CAPM be interpreted as a special case of the Arbitrage Pricing Theorem? [3 marks]

(iii) Three well diversified portfolios A, B, C each satisfy the following multi-factor model (for  $i = A, B, C$ ):  
 $R_i = a_i + b_{i1}I_1 + b_{i2}I_2$

(a) Find the equation of the plane of all diversified portfolios under the Arbitrage Pricing Theory given the following Data

<u>Portfolio</u>	<u>Expected Return</u>	$b_{i1}$	$b_{i2}$
A	10%	1	1
B	9%	1	0,5
C	8%	0,5	1

**[10 marks]**

(b) Find the risk-free rate of interest. [2 marks]

**[Total 23 marks]**

5. State the efficient market hypothesis. [4 marks]

---

**END OF EXAMINATION PAPER**