NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF COMMERCE

DEPARTMENT OF INSURANCE AND ACTUARIAL SCIENCE

B.COMM (HONOURS) DEGREE IN ACTUARIAL SCIENCE

CIN 4115 – FINANCIAL ECONOMICS

APRIL / MAY 2009-FIRST SEMESTER EXAMINATIONS

DURATION: 3 HOURS

Instructions to Candidates

1. **Attempt all 9 Questions**
2. **To Obtain Full Marks Show ALL appropriate steps to your answers**

Requirements

1. **Actuarial Tables (2002) Edition**
2. **Non-programmable Scientific calculator**

P1. Describe briefly the meaning of:

(i)	Shortfall Probability	(1 mark)
(ii)	Indifference curves	(2 marks)
(iii)	Karush – Kuhn – Tucker Conditions (KKT)	(4 marks)

[Total 7 marks]

P2.

(i) State the expected utility theorem and the characteristic of non-satiation. (3 marks)

(ii) A consultant, who is risk-averse and non-satiated, has been offered a one year contract with profit-sharing arrangements. His basic fee will be \$80,000. In addition, he will be paid an extra \$20,000 if the company profits exceed a certain target. The probability of the company making enough profits is 0.65.

(a) His utility function is of the form $U(w) = w - 0.4w^2$ (w is expressed as a proportion of \$100,000). Derive the remuneration range over which $U(w)$ gives an appropriate representation of his individual preferences . (3 marks)
(b) Calculate the expected total remuneration and the expected utility offered by the job. (2 marks)

(c) If he is to be offered the alternative of a fixed fee, calculate the minimum he should accept. (4 marks)

[Total 12 marks]

P3. Investment projects A and B have both a rate of return that is normally distributed with expected return μ. The variance of the rate of return of project A is σ_A^2, the variance of the rate of return of project B is σ_B^2, with $\sigma_A^2 < \sigma_B^2$. Show that:

(i) A does not dominate B with respect to first order stochastic dominance.
(4 marks)

(ii) A does not dominate B with respect to third order stochastic dominance.
(5 marks)

[Total 9 marks]

P4. Consider the following returns-generating model

$$R_i = r_f + \beta_i (R_M - r_f) + \sigma_i \varepsilon_i \text{ for } i = 1 \ldots n$$

Where:

R_i = random variable representing the rate of return on security i
r_f = (known) rate of return on a risk-free asset
R_M = random variable representing the contemporaneous rate of return on the market index
ε_i = standard normal random variable (that is independent of R_M and of ε_j for each j distinct from i)
β_i,σ_i are unknown constants specific to security i.

(i) Explain, in detail, how the above model considerably simplifies the estimation problems in mean-variance portfolio modelling when n is large. (3 marks)

(ii) Set down the equations for the expected returns based on security i for the Capital Asset Pricing Model (CAPM) and the Arbitrage Pricing Theory (APT). Define all symbols used. (3 marks)

(iii) Briefly explain the major differences between these models. (5 marks)

(iv) State the main assumptions needed in each case to ensure consistency between the returns-generating model in (i) and the CAPM and the APT.
(2 marks)

(v) An investor can invest (long or short) in only two assets with annual returns R_A and R_B with the following (annualised) characteristics:

Asset	*Expected rate of return*	*Standard deviation*
A	0.15	0.2
B	0.1	0.1

The correlation between the returns on the assets is 0.5.
No adjustments to the investor s portfolio are possible within the year.

(a) Determine the global minimum variance portfolio proportions. (3 marks)

(b) Derive expressions for both the expected return and standard deviation of a general portfolio on the efficient frontier in terms of a common parameter and hence explain how the efficient frontier can be derived. (3 marks)

(vi) The market is now expanded by the addition of a risk free asset with annualised rate of return of 7%.Derive the equation for the new efficient frontier. (4 marks)

[Total 23 marks]

P6. You are given the following summary statistics for monthly returns on 3 stocks and the S&P Index (corrected for dividends) for a 12-month period:

	A	*B*	*C*	*S&P*
		Security		
		S_x		
	35.35	72.37	42.65	36.06
		S_{xy}		
A	613.8439	221.5418	739.4184	296.9104
B		559.2715	649.0168	256.0504
C			3179.835	582.4529
S&P				250.8953

where S_x is the sum of the 12 monthly returns for each security and S_{xy} is as defined in the Linear Regression section of the *Formulae and Tables for Actuarial Examinations.*

(i) Compute the mean return and variance of return for each stock using:

(a) the single-index model
(b) the historical data. (4 marks)

(ii) Compute the covariance between each possible pair of stocks using:

(a) the single-index model
(b) the historical data. (5 marks)

(iii) Compute the return and standard deviation of a portfolio constructed by placing one-third of your funds in each stock using:

(a) the single-index model
(b) the historical data. (6 marks)

(iv) Explain why the answers to (i) (a) and (i) (b) were the same while the answers to (ii) (a) and (ii) (b) and (iii) (a) and (iii) (b) were different. (3 marks)

[Total 18 marks]

P7.

(i) Describe informational efficiency in the context of the Efficient Markets Hypothesis. (4 marks)

(ii) Briefly outline three examples of effects that have been claimed to exist in stock markets that might be considered examples of informational inefficiency. (3 marks)

(iii) List the reasons, with brief explanations, why it is difficult to assess empirically whether or not the market is efficient. (4 marks)

[Total 11 marks]

P8. You consider buying shares of Company A and of Company B. Your investment decision is based on a two–index model. The return on the stock of A is given by:

$R_A = 1.0 + 0.0I_L^{/} + 0.5I_S^{/} + C_A^{/}$ and for B by $R_B = 2.0 + 0.8I_L^{/} + 0.0I_S^{/} + C_B^{/}$

where

$I_L^{/}$ denotes the return on an index of large stocks

$I_S^{/}$ denotes the return on an index of small stocks

Assume that:

$C_A^{/}$ and $C_B^{/}$ are uncorrelated and have zero mean.

$C_i^{/}$ and $I_L^{/}$ are uncorrelated, $i \in \{A, B\}$

$C_i^{/}$ and $I_S^{/}$ are uncorrelated, $i \in \{A, B\}$

(a) Regression analysis shows that $I_S^{/}$ is related to $I_L^{/}$ via

$I_S^{/} = 1.0 + 1.5\, I_L^{/} + d_t$

where d_t and $I_L^{/}$ are uncorrelated.

Express the returns on the stocks of A and on the stocks of B in a transformed two–index model with orthogonal indices. (8 marks)

(b) Calculate the mean and variance for each stock given the following data:

$E[I_L] = 8\%$

$E[I_S] = 10\%$

$\sigma_L^{\ 2} = 5\%\%$

$\sigma_s^2 = 8\%\%$

$\sigma_c^2 A = 12\%\%$

$\sigma_c^2 B = 10\%\%$

Where I_L, I_S denotes the orthogonal indices with variance σ_L^2 *and* $\sigma_S^{\ 2}$ respectively. σ_{ci}^2 denotes the variance of $C_i^{/}$, for $i \in \{A, B\}$

(5 marks)

[Total: 13 marks]

P9. Consider four assets with expected return $\mu_1 = 6\%,\ \mu_2 = 7\%, \mu_3 = 8\%$ and $\mu_4 = 10\%$ with the following variance – covariance matrix (Units are %%).

$$\begin{matrix} 10 & 8 & 0 & 15 \\ 8 & 15 & 0 & 16 \\ 0 & 0 & 40 & 0 \\ 15 & 16 & 0 & 35 \end{matrix}$$

An investor wants to calculate the minimum variance portfolio for a given expected return E_P. He or she expresses this problem in matrix notation as $Ay = b$. Write down the matrices $A, y\ and\ b$.

[Total 6 marks]

**********************END OF EXAMINATION**********************