

NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

B.COMM (HONOURS) DEGREE IN ACTUARIAL SCIENCE

ACTUARIAL MATHEMATICS IIB – CIN 4210

MAY/JUNE 2005 SECOND SEMESTER EXAMINATION

DURATION: 3 HOURS

Instructions to Candidates

1. Write your student number on the answer booklet
2. Begin each question on a separate sheet.
3. Marks to each question are shown in brackets
4. Attempt all 15 questions

**ADDITIONAL MATERIAL**

An electronic calculator

A copy of the Actuarial Examination Tables

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- Q1. Under the Manchester Unity model of sickness, you are given the following values

$$s_x = 5$$

$$\int {}_tP_x dt = 0.9$$

Calculate the value of  $z_x$ . **[2 marks]**

- Q2. Give a formula for  $P_{21}(2003)$  in terms of  $P_{20}(2002)$ , based on the component method of population projection.  $P_x(n)$  denotes the population aged  $x$  last birthday at mid-year  $n$ .

State all the assumptions that you make and define carefully all the symbols that you use. **[3 marks]**

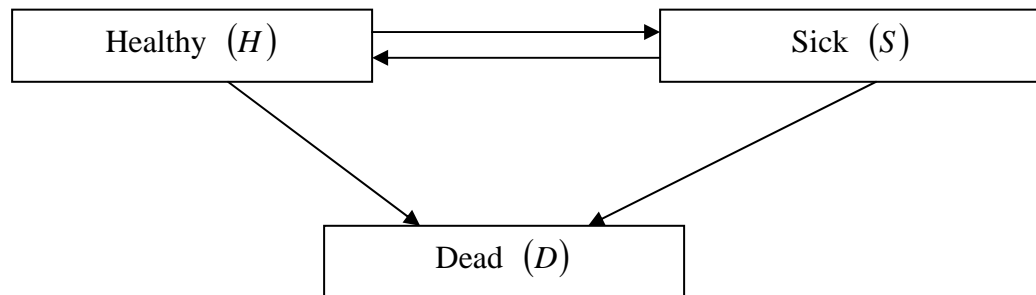
- Q3. A life insurance company issues a policy under which sickness benefit of £100 per week is payable during all periods of sickness. There is a waiting period of 1 year under the policy.

You have been asked to calculate the premium for a life aged exactly 30, who is in good health, using the Manchester Unity model of sickness.

Describe how you would allow for the waiting period in your calculation, giving a reason for your choice of method. **[3 marks]**

- Q4. An employer recruits lives aged exactly 20, all of whom are healthy when recruited. On entry, the lives join a scheme that pays a lump sum of £50,000 immediately on death, with an additional £25,000 if the deceased was sick at the time of death.

The morality and sickness of the scheme members are described by the following multi-state model, in which the forces of transition depend on age only.



All surviving members retire at age 65 and leave the scheme regardless of their state of health.

$p_{x,t}^{ab}$  is defined as the probability that a life who is in state  $a$  ( $a = H, S, D$ ) at age  $x$  is in state  $b$  at age  $x + t$  ( $t \geq 0$  and  $b = H, S, D$ ).

Write down an integral expression for the expected present value, at force of interest  $\delta$ , of the death benefit in respect of a single new recruit. **[3 marks]**

- Q5. A pension scheme provides a pension of  $\frac{1}{60}$  of career average salary in respect of each full year of service, on age retirement between the ages of 60 and 65. A proportionate amount is provided in respect of an incomplete year of service.

At the valuation date of the scheme, a new member aged exactly 40 has an annual rate of salary of £40,000.

Calculate the expected present value of the future service pension on age retirement in respect of this member, using the Pension Fund Tables in the Formula and Tables for Actuarial Examinations. **[3 marks]**

- Q6. Outline the problems associated with investment policy if guaranteed surrender values are offered on conventional policies. **[6 arks]**

- Q7. The staff of a company is subject to two modes of decrement, death and withdrawal from employment.

Decrements due to death take place uniformly over the year of age in the associated single-decrement table: 50% of the decrements due to withdrawal occur uniformly over the year of age and the balance occurs at the end of the year of age, in the associated single-decrement table.

You are given that the independent ratio of mortality is 0.001 per year of age and the independent rate of withdrawal is 0.1 per year of age.

Calculate the probability that a new employee aged exactly 20 will die as an employee at age 21 last birthday. **[5 marks]**

- Q8. The following data are available from a life insurance company relating to the mortality experience of its temporary assurance policy holders.

$\theta_{x,d}$  The number of deaths over the period 1 January 1998 to 30 June 2001, aged  $x$  nearest birthday at entry and having duration  $d$  at the policy anniversary next following the date of death.

$P_{y,e}(n)$  The number of policyholders with policies in force at time  $n$ , aged  $y$  nearest birthday at entry and having curtate duration  $e$  at time  $n$ , where  $n = 1.1.1998, 30.6.1998, 30.6.2000$  and  $30.6.2001$ .

Develop formulae for the calculation of the crude central select rates of mortality corresponding to the  $\theta_{x,d}$  deaths and derive the age and duration to which these rates apply. State all the assumptions that you make. **[8 marks]**

- Q9. Outline the effects of occupation and nutrition on mortality. **[4marks]**

- Q10. A twenty – year endowment insurance with sum assured £75,000 payable at end of year of death, or on maturity, is issued to a policyholder aged 40. The premiums were set assuming initial expenses of 75% of the first year’s premium and 5% of all subsequent premiums. After 8 years the policy holder decides to make the policy paid – up. By considering the realistic gross premium reserves, calculate the revised sum assured. The renewal expense of 5% ceases on cessation of premium payment, although there will be administration expenses after going paid-up of £50 pa.

Basis: AM 92 ultimate, 4% interest, alteration expenses £75. **[8 marks]**

- Q11. A life insurance company issues 2-year without-profit policies to the employees of a certain company, providing the following benefits:

- (a) on death during the 2 years, a sum of £10,000:
- (b) on withdrawal from the company within 2 years, a return of premiums paid without interest:
- (c) on survival as an employee of the company to the end of 2 years, the sum of £500.

All benefits are paid at maturity or at the end of the year of claim, if earlier. There are no decrements from service other than those mentioned.

Calculate the level annual premium for the above policy issued to an employee aged exactly 50. Premiums are payable annually in advance for 2 years so long as the employee is in the service of the company at the dates at which premiums are due.

Basis:

mortality	the independent rate of mortality is that of A1967-70 Ultimate
withdrawal	the independent rate of withdrawal is 0.05 at ages 50 and 51
interest	5% per annum
expenses	ignored

**[Total: 9 marks]**

- Q12. A pension scheme provides a pension of  $1/60^{\text{th}}$  of final pensionable salary for each year of service (with part years counting proportionately) payable on normal retirement at age 65 or on voluntary early retirement before age 65. Final pensionable salary is defined to be the average pensionable salary during the 36 months immediately prior to retirement. Pensionable salary is defined to be the annual rate of salary less a fixed deduction of £2,000.

No benefit is payable on withdrawal or ill-health retirement.

Members' contributions of 5% of pensionable salary are deducted from members' monthly pay. Pay levels are reviewed on 1 January each year.

A group of 5 men all aged 35 nearest birthday joined the scheme (as part of a transfer from another company) on 1 May 1998. The total salary these members would have received during the year ending on 30 April 1998, if they had been working for the company during that period, would have been £75,000.

The trustees have asked the employer to meet the cost of future benefits for these members by making monthly contributions proportional to the amounts contributed by the members throughout the remainder of their service with the company.

- (i) Calculate the required employer's contribution rate for these members, assuming that the service table, salary scale, interest rate and other actuarial assumptions used in the Tables are appropriate in this case. **[9 marks]**
- (ii) Outline how you would modify your calculations if
- the £75,000 referred to the members' starting salaries on 1 May 1998 (with subsequent pay reviews on 1 January each year).
  - the period of service counting in the benefit formula was limited to a maximum of 20 years.

(You are not required to give a full set of formulae.) **[4 marks]**  
**[Total: 12 marks]**

- Q13. (i) You are provided with the following results for two occupations A and B: **[7 marks]**

	All Occupations		Occupation A		Occupation B	
<i>Age group</i>	<i>Population at risk (000s)</i>	<i>Deaths</i>	<i>Population at risk (000s)</i>	<i>Deaths</i>	<i>Population at risk (000s)</i>	<i>Deaths</i>
16-34	360	360	21	21	12	36
35-44	390	780	42	84	44	88
45-54	430	2580	93	372	92	460
55-64	320	7680	78	2028	72	1512
Total	1500	11400	234	2505	220	2096

Calculate the crude death rates, standardised mortality rates and standardised mortality ratios for occupations A and B using the 'All Occupations' experience as standard.

- (ii) It has been suggested that administration of the investigation would be simplified if, for the individual occupations, data on ages were supplied only for deaths. Within each group, the actual deaths and the “All Occupations” mortality rate can be used to estimate the population. The

ratio of total population estimated by this method to total actual population in each occupation provides an alternative index for the mortality experience.

Using the data in (i), calculate the value of this new index for each occupation, and comment on the results obtained in parts (i) and (ii). **[7 marks]**

**[Total: 14 marks]**

- Q14. (i) Explain briefly the terms ‘waiting period’ and ‘off period’ used in connection with sickness benefits. **[3 marks]**
- (iii) Members of a friendly society pay a level contribution which ceases at age 65 and is waived during periods of sickness. The benefits are as follows:
- (a) a sickness benefit of £50 per week for the first 26 weeks of sickness, £20 thereafter, the benefit ceasing at age 65,
  - (b) immediately on death before age 65, a lump sum of £10,000 plus a return of contributions (including those waived during sickness) without interest,
  - (c) on survival to age 65, an annuity of £3,000 per annum payable by monthly instalments in advance for 5 years and throughout life thereafter.

There is no waiting period for the sickness benefit, and the off period may be assumed to be the same as that underlying the *Tables*.

Calculate the weekly contribution payable by a member aged 30 exactly at Entry.

Basis: Mortality - Before age 65: ELT15 (Males)  
 After age 65: PMA 92C20  
 Sickness- S(MU)  
 Interest- 4% - *pa*

Assume that  $(IA)_{30:35}^1 = 1.7$  according to this basis. **[14 marks]**

**[Total: 17 marks]**

- Q15. A life office issues a special ten year policy to a life aged exactly 55 under which the annual premium increases by £100 each year. The sum assured payable on death within ten years is £20,000 payable at the end of the year of death. On

survival to the end of the ten years term the policy holder receives a refund of all the premiums paid, without interest.

- (i) Show that the first premium payable is £552. **[8 marks]**
- (ii) Immediately before the sixth premium is paid the policy holder requests that the future premiums remain constant at the same level as the fifth premium, resulting in a lower refund of premiums at the end of the term.

If the expense of alteration to the office is £100, calculate the revised sum assured payable on death in last five years. **[11 marks]**

Basis for premiums and alteration:

Mortality: AM92 ultimate

Interest: 4%

Expenses: initial £150

Plus 4% of each premium including the first.

**[Total: 19 marks]**

**END OF EXAMINATION**