

**NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY**

**Bachelor Of Commerce (Actuarial Science) Honours Degree**

**ACTUARIAL MATHEMATICS IIB – CIN 4210**

**April/May 2006 Second Semester Examination**

**Duration: 3 Hours**

***Instructions To Candidates***

- 1. Write your student number on the answer booklet.***
- 2. Begin each question on a separate sheet.***
- 3. Marks to each question are shown in brackets***
- 4. Attempt all 13 questions***

***Additional Material***

- 1. An electronic calculator.***
- 2. Two copies of Actuarial Examinations Tables.***

**Question One**

Define the term asset share in the context of a with-profit policy. **[4 Marks]**

**Question Two**

A pension scheme provides a pension on ill-health retirement of  $1/80^{\text{th}}$  of Final Pensionable Salary for each year of pensionable service subject to a maximum pension of  $20/80^{\text{th}}$  of Final Pensionable Salary. Final Pensionable Salary is defined as the average salary earned in the three years before retirement. Normal retirement age is 65 exact.

Defining all terms used, derive a formula for the present value of the ill-health retirement benefit for a member currently aged 35 exact with exactly 10 years past service and salary for the year before the calculation date of \$20,000. **[7 Marks]**

**Question Three**

List the main assumptions used in valuing the benefits of a defined benefit occupational pension scheme. **[5 Marks]**

#### **Question Four**

Describe how occupation affects morbidity and mortality.

**[4 Marks]**

#### **Question Five**

Explain how an insurance company uses risk classification to control the profitability of its life insurance business.

**[5 Marks]**

#### **Question Six**

Due to a downturn in the economy, the numbers unemployed in a certain country are expected to increase. The current number unemployed is 100,000, and this is expected to rise towards but not exceed 300,000 following the logistic growth model. The initial rate of increase in unemployed will be 25% p.a. Calculate how long it will take for the unemployed population to reach 200,000.

**[7 Marks]**

#### **Question Seven**

A population is subject to two modes of decrement,  $\alpha$  and  $\beta$ , between ages  $x$  and  $x+1$ . In the single decrement tables

$${}_t p_x^\alpha = (x / x+1)^2$$

and,

$${}_t p_x^\beta = (x / x+1)^3$$

where;  $0 \leq t \leq 1$ .

Write down an integral expression for  $(aq)_x^\alpha$ . Hence or otherwise obtain an expression for this probability in terms of  $x$  only.

**[6 Marks]**

#### **Question Eight**

- i. Give a formula for the area comparability factor defining all terms you use.
- ii. Explain the role of this Factor in calculating the standardized mortality rates, indicating any advantages it has over other available methods.

**[4 Marks]**

### **Question Nine**

On 1 June 1997 a man is aged 34 exactly. The salary scale given below is such that for an individual aged exactly  $x$  on 1 June and for any integer  $t > 0$ :

$$\frac{S_{x+t}}{S_x} = \frac{\text{Expected earnings between ages } x+t \text{ and } x+t+1}{\text{Expected earnings between ages } x \text{ and } x+1}$$

Final salary is defined as the average earnings over the three years immediately prior to retirement. Salaries are increased annually on 1 December. On 1 December 1996 this member's salary was increased to \$23,000 p.a.

<b>Exact age (<math>x</math>)</b>	<b>Salary scale (<math>S_x</math>)</b>
32	1.54
33	1.62
34	1.73
35	1.85
36	1.96
.	.
.	.
.	.
58	4.55
59	4.63
60	4.7
61	4.76
62	4.81
63	4.85
64	4.88

Calculate the expected final salary to the nearest \$100 of this member on the basis that he will retire between ages 62 and 63 (and is equally likely to retire at any point in this age range). **[7 Marks]**

### **Question Ten**

- i. Discuss the suitability of the crude death rate, the standardized mortality rate and the standardized mortality ratio for comparing:
- (a). The mortality, at different times, of the population of a given country  
(b). The mortality, at a certain time, of two different occupational groups in the same population. **[6 Marks]**
- ii. The following table gives a summary of mortality for one of the occupational groups and for the country as a whole.

	<b>Occupation A</b>		<b>Whole Country</b>	
<b>Age group</b>	<b>Exposed to risk</b>	<b>Deaths</b>	<b>Exposed to risk</b>	<b>Deaths</b>
20 - 34	15,000	52	960,000	3,100
35 - 49	12,000	74	1,400,000	7,500
50 - 64	10,000	109	740,000	7,100
<b>Total</b>	<b>37,000</b>	<b>235</b>	<b>3,100,000</b>	<b>17,700</b>

Calculate the crude death rate, the standardized mortality rate and the standardized mortality ratio for Occupation A. **[6 Marks]**  
**[Total 12 Marks]**

### **Question Eleven**

- i. Define the following terms in the context of an application for a temporary life assurance contract, and give an example of each:
- a. Temporary initial selection  
b. Self-selection **[3 Marks]**
- ii. List the topics that you would expect to be included in the questions on a proposal form, which a life company might use for risk classification purposes. The contracts concerned are five-year temporary assurance policies and the expected applicants are men aged 25-45. **[4 Marks]**

**[Total 7 Marks]**

### **Question Twelve**

A life aged 40 purchases a 25-year endowment assurance contract. Level quarterly premiums are payable throughout the duration of the contract. The sum assured of \$100,000 is payable at maturity or at the end of the year of death.

- i. Show that the quarterly premium is \$704.61. [6 Marks]

Basis	
Mortality	A1967-70 Select
Interest	4% p.a.
Initial expenses	\$250 plus 60% of the gross annual premium
Renewal expenses	3% of the second and subsequent quarterly premiums
Claims expenses	\$500 on death; \$100 on maturity

- ii. At age 60, immediately before the premium then due, the life wishes to surrender the policy. The insurer calculates a surrender value equal to the gross retrospective policy value, assuming the same basis as in (i), but with 3% p.a. interest.
- a. Calculate the surrender value.
- b. Explain with reasons whether the surrender value would have been larger, the same or smaller than in (ii.a) if the office had used the prospective gross premium policy value, on the same basis. [9 Marks]

[Total 15 Marks]

**Question Thirteen**

A life office issues policies to lives age 58 exact. These policies provide the following benefits:

- i. On becoming permanently disabled before the 60<sup>th</sup> birthday, an annuity of \$5,000 p.a. payable weekly for life with a lump sum of \$100,000 on death, and
- ii. On death before the 60<sup>th</sup> birthday, if not previously permanently disabled, a lump sum of \$50,000 if death was due to accidental causes.

In either case the death benefit is payable immediately on death. Calculate the annual amount of premium, payable weekly and ceasing on death, permanent disability or the 60<sup>th</sup> birthday, on the following basis:

Mortality: The independent rates of mortality of lives that are not permanently disabled are those of the A1967-70 Ultimate table; 6% of these deaths are due to accidental causes.

The permanently disabled are subject to the mortality of the English Life Table No 12 Males with the age rated up by 10 years.

Permanent disability: A constant independent rate of 0.007

All decrements are assumed to be uniformly distributed over each year of age.

Interest: 4% per annum

Expenses: 5% of all premiums

**[Total 17 Marks]**

**\*\*\*\*\* END OF EXAMINATION \*\*\*\*\***