

**NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY**

**Bachelor Of Commerce (Actuarial Science) Honours Degree**

**ACTUARIAL MATHEMATICS IIB – CIN 4210**

**July 2006 Supplementary Examination**

**Duration: 3 Hours**

***Instructions To Candidates***

- 1. Write your student number on the answer booklet.***
- 2. Begin each question on a separate sheet.***
- 3. Marks to each question are shown in brackets***
- 4. Attempt all 11 questions***

***Additional Material***

- 1. An electronic calculator.***
- 2. Two copies of Actuarial Examinations Tables.***

**Question One**

Describe the calculation of a surrender value for a without profit endowment assurance policy, under which level annual premiums are payable monthly in advance and cease on earlier death or surrender and the sum assured is payable immediately on death. Give formulae, defining carefully all the symbols that you use.

**[5 Marks]**

**Question Two**

Define each of the following terms and give one example of each:

- i. Class selection**
- ii. Selective decrement**
- iii. Spurious selection**

**[6 Marks]**

**Question Three**

Describe four benefit options that may be available to an individual member of a pension scheme who leaves the scheme before normal pension age.

**[4 Marks]**

#### **Question Four**

A pension scheme provides an ill-health retirement pension of  $1/60$  of Final Pensionable Salary for each year of complete service, with fractions of a year to count proportionately, subject to a maximum pension of  $40/60$  of Final Pensionable Salary. Retirement due to ill health may take place at any age before age 65. Final Pensionable Salary is defined as the average annual salary over the three- year period preceding retirement.

Derive commutation functions to value the ill-health retirement pension for a member aged exactly 25, who has completed exactly 5 years company service to date. Define carefully all the symbols that you use.

**[7 Marks]**

#### **Question Five**

- i. In the context of a pension scheme, explain the term “prospective service benefit” and state one example.

**[3 Marks]**

- ii. Mortality levels for a certain country have been studied at national and regional level. Explain the circumstances under which a particular region may have an Area Comparability Factor of 0.5.

**[3 Marks]**

**[Total 6 Marks]**

#### **Question Six**

- i. Define the term Total Fertility Rate and explain the difference between rates calculated on a cohort basis and a period basis.

**[3 Marks]**

- ii. In the context of population projection, state, with a reason, which basis is preferable.

**[3 Marks]**

**[Total 6 Marks]**

### **Question Seven**

A pension fund provides a pension on age retirement of one-sixtieth of final salary for each year of service while a member of the scheme. Final salary is defined as the average earnings over the three years prior to retirement. Members contribute at the rate of 5% of salary per annum towards the cost of their benefits.

One member aged 40 exact has 10 years of past service and earned \$35,000 over the last year. Benefits are valued according to the assumptions in the Pension Fund Tables in the Formulae and Tables for Actuarial Examinations.

- i. Calculate the past service liability for this member.  
**[3 Marks]**
- ii. Calculate the expected present value of the member's future contributions up to date of exit.  
**[3 Marks]**

**[Total 6 Marks]**

### **Question Eight**

A life insurance company offers an option on its 10-year without profit term assurance policies to effect a whole life without profits policy, at the expiry of the 10-year term, for the then existing sum assured, without evidence of health. Premiums under the whole life policy are payable annually in advance for the whole of life, or until earlier death.

- i. Describe the conventional method of pricing this option, stating clearly the data assumptions required. Formulae are not required.  
**[4 Marks]**
- ii. A policyholder aged exactly 30 wishes to effect a 10-year without profits term assurance policy, for a sum assured of \$100,000. Calculate the additional single premium, payable at the outset, for the option, using the conventional method.

The following basis is used to calculate the basic premiums for the term assurance policies.

Mortality: A1967-70 Select

Interest: 6% p.a.

Expenses: none

**[5 Marks]**

- iii. Describe how you would calculate the option single premium for the policy described in part (ii) above using the North American method, stating clearly what additional data you would require and what assumptions you would make.  
**[4 Marks]**

- iv. State, with reasons, whether it would be preferable to use the conventional method or the North American method for pricing the mortality option under the policy described in part (ii) above.  
**[3 Marks]**

**[Total 16 Marks]**

**Question Nine**

Write down formulae that could be used to project the population in a well-developed country, defining all symbols used.

**[10 Marks]**

**Question Ten**

A non-profit endowment assurance policy with a term of 30 years is to be issued to a life aged 35 exact. The sum assured is \$40,000 and will be paid immediately upon death or upon survival to age 65. Level premiums are payable weekly in advance for 20 years or until the death of the policyholder, if earlier.

The policy includes a premium waiver clause covering periods during which the policyholder is ill. Under this clause the policyholder must pay premiums for the first 12 months of sickness. If sickness persists for more than 12 months then premiums are waived until the policyholder recovers.

Calculate the amount of the weekly premium for this policy using the following basis:

Interest	4% per annum
Mortality	English Life Table Number 12 – Males
Sickness	Manchester Unity Sickness Experience 1893-97 Occupational group A, H, J
Expenses: Initial	\$100
Weekly	2.5% of each premium including the first, incurred at each premium payment date. Weekly expenses are not incurred during periods when the premium is being waived.

**[Total 15 Marks]**

**Question Eleven**

- i. On 1 September 1996, a life aged exactly 50 purchased a deferred annuity policy, under which yearly benefit payments are to be made. The first payment, being \$10,000, is to be made at age exact 60 if he is then alive. The payments will continue yearly during his lifetime, increasing by 1.923% p.a. compound. Premiums under the policy are payable annually in advance for 10 years or until earlier death.

If death occurs before age 60, the total premiums paid under the policy, accumulated to the end of the year of death at a rate of interest of 1.923% p.a. compound, are payable at the end of the year of death. Calculate the annual premium.

**[10 Marks]**

<b>Basis</b>		
Mortality	Before age 60	A1967-70 Ultimate
	After age 60	a(55) Males Ultimate
Interest		6% p.a.
Expenses	Initial	10% of the initial premium, incurred at the outset
	Renewal	5% of each of the second and subsequent premiums, payable at the time of premium payment
	Claim	\$100, incurred at the time of payment of the death benefit

- ii. On 1 September 2001, immediately before payment of the premium then due, the policyholder requests that the policy be altered so that there is no benefit payable on death and the rate of increase for the annuity in payment is to be altered. The premium under the policy is to remain unchanged, as is the amount of the initial annuity payment.

A life insurance company calculates the revised terms of the policy by equating gross premium prospective reserves immediately before and after alteration, calculated on the original pricing basis, allowing for an expense of alteration of \$100.

Calculate the revised rate of increase in payment of the annuity.

**[9 Marks]**

**[Total 19 Marks]**

\*\*\*\*\* **END OF EXAMINATION** \*\*\*\*\*