NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF COMMERCE

DEPARTMENT OF INSURANCE AND ACTUARIAL SCIENCE

B.COMM (HONOURS) DEGREE IN ACTUARIAL SCIENCE

CIN 4210- ACTUARIAL MATHEMATICS IIB

AUGUST 2011 SUPPLIMENTARY EXAMINATIONS

Time allowed: Three hours

Requirements

1. **Actuarial Tables (2002) Edition**
2. **Non-programmable Scientific calculator**

INSTRUCTIONS TO THE CANDIDATE

1. Enter all the candidate and examination details as requested on the front of your answer booklet.

2. You must not start writing your answers in the booklet until instructed to do so by the supervisor.

3. Mark allocations are shown in brackets.

4. Attempt all 16 questions, beginning your answer to each question on a separate sheet.

5. Candidates should show calculations where this is appropriate.

Graph paper is not required for this paper.

1) Explain why a life insurance company will need to set up reserves for the endowment assurance contracts it has sold. [3]

2) A life insurance company issues an annuity to a life aged 60 exact. The purchase price is £200,000. The annuity is payable monthly in advance and is guaranteed to be paid for a period of 10 years and for the whole of life thereafter. Calculate the annual annuity payment.

   Basis:

Mortality AM92 Ultimate
Interest 6% per annum [4]

3) A twelve-year life insurance contract has the following profit signature before anynon-unit reserves are created:
(+1, -1, +1, +1, +1, -1, 0, -1, +1, -1, +1, +1)
Non-unit reserves are to be set up to zeroise the negative cash flows. Write down the revised profit signature, ignoring interest. [3]

4) A life insurance company issues a reversionary annuity contract. Under the contract an annuity of £20,000 per annum is payable monthly for life, to a female life now aged 60 exact, on the death of a male life now aged 65 exact. Annuity payments are always on monthly anniversaries of the date of issue of the contract. Premiums are to be paid monthly until the annuity commences or the risk ceases.

Calculate the monthly premium required for the contract.

Basis: Mortality: PFA92C20 for the female
PMA92C20 for the male
Interest: 4% per annum
Expenses: 5% of each premium payment&1.5% of each annuity payment [6]

5) Explain, in the context of the lapse rates of life insurance policies, what is meant by:

(a) Class selection
(b) Temporary initial selection
(c) Time selection

Give an example in each case. [6]

6) A life office has issued for a number of years whole-life regular premium policies toagroup of lives through direct advertising. Assured lives are only required to completean application form with no further evidence of health. Outline the forms of selectionthat the insurer should expect to find in the mortality experience of the lives. [6]

7) A member of a pension scheme is aged 55 exact, and joined the scheme at age 35exact. She earned a salary of £40,000 in the 12 months preceding the scheme

valuation date. The scheme provides a pension on retirement for any reason of 1/80th of final pensionable salary for each year of service, with fractions countingproportionately. Final pensionable salary is defined as the average salary over the three years prior to retirement. Using the functions and symbols defined in, and assumptions underlying, the Example Pension Scheme Table in the Actuarial Tables:

(i) Calculate the expected present value now of this member s total pension. [4]

(ii) Calculate the contribution rate required, as a percentage of salary, to fund the future service element of the pension. [2]

[Total 6]

8)A male life aged 60 exact wants to buy the following benefits within one policy:

(a) An annuity of £5,000 per annum payable monthly in arrear to his wifecurrently aged 55 exact commencing on his death and for the rest of her life, and

(b) an annuity of £2,000 per annum payable monthly in arrear to his grandsoncurrently aged 13 exact commencing on the death of either grandparent andceasingwhen the grandson reaches age 21.

Calculate the overall single premium.

Basis:
Mortality Male life – PMA92C20
Wife – PFA92C20
Grandson – ignore
Interest 4% per annum [10]

9) (i) Calculate the expected present value of an annuity-due of 1 per annum payable annually in advance until the death of the last survivor of two lives using thefollowing basis:
First life: male aged 70, mortality table PMA92C20
Second life: female aged 67, mortality table PFA92C20
Rate of interest: 4% per annum [2]

(ii) Give an expression for the variance of the annuity-due in terms of annuityfunctions.
[5]

[Total 7]

10) Let X be a random variable representing the present value of the benefits of a pureendowment contract and Y be a random variable representing the present value of thebenefits of a term assurance contract which pays the death benefit at the end of theyear of death. Both contracts have unit sum assured, a term of n years and wereissued to the same life aged x.

(i) Derive and simplify as far as possible using standard actuarial notation anexpression for the covariance of X and Y. [4]

(ii) Hence or otherwise, derive an expression for the variance of (X+Y) and simplify it as far as possible using standard actuarial notation. [4]

[Total 8]

11) (i) Explain the terms "unit fund" and "non-unit fund" in the context of a unitlinkedlife assurance contract. [4]

(ii) Explain why a life insurance company might need to set up reserves in orderto zeroise future expected negative cashflows in respect of a unit-linked lifeassurance contract. [2]

(iii) A life insurance company issues 4-year unit-linked contracts to a male livesaged 50 exact. The following non-unit fund cash flows, NUCFt, (t = 1, 2, 3, 4) are obtained at the end of each year t per contract in force at the start of the year t:

Year t	1	2	3	4
NUCFt	375.4	-152.0	-136.2	-118.0

The rate of interest earned on non-unit reserves is 5.5% per annum and mortality follows the AM92 Select table.

Calculate the reserves required at times t = 1, 2 and 3 in order to zeroise future negative cash flows. [4]

[Total 10]

12) An employer wishes to introduce a lump-sum retirement benefit payable immediately on retirement at 65 or earlier other than on the grounds of ill-health. The amount of the benefit is £1,000 for each year of an employee s service, with proportionate partsof a year counting.

(i) Give a formula to value this benefit for an employee currently aged x with nyears of past service, defining all terms used. [5]

(ii) Using the Pension Scheme Tables from the Actuarial Formulae and Tables,calculate the value for an employee currently aged 30 exact with exactly 10 years past service.

[2]

(iii) Calculate the level annual contribution payable continuously throughout thisemployee s service to fund the future retirement benefit.

[3]

[Total 10]

13)Two lives, a female aged 60 exact and a male aged 65 exact, purchase a policy withthe following benefits:

(i) An annuity deferred ten years, with £20,000 payable annually in advance for aslong as either of them is alive

(ii) A lump sum of £100,000 payable at the end of the policy year of the first death,should this occur during the deferred period

Level premiums are payable monthly in advance throughout the deferred period or until earlier payment of the death benefit.

Calculate the monthly premium.

Basis:
Mortality Female PFA92C20
Male PMA92C20
Interest 4% per annum
Expenses Initial £350
Renewal 2.5% of each monthly premium excluding the first. [14]

14) A life insurance company issues a 4-year unit-linked endowment assurance contract to a male life aged 40 exact under which level premiums of £1,000 per annum are payable in advance. In the first year, 50% of the premium is allocated to units and 102.5% in the second and subsequent years. The units are subject to a bid-offer spread of 5% and an annual management charge of 0.5% of the bid value of the units is deducted at the end of each year. If the policyholder dies during the term of the policy, the death benefit of £4,000 or the bid value of the units after the deduction of the management charge, whichever is higher, is payable at the end of the year of death. On surrender or on survival to the end of the term, the bid value of the units is payable at the end of the year of exit. The company uses the following assumptions in its profit test of this contract:

Rate of growth on assets in the unit fund 6% per annum
Rate of interest on non-unit fund cashflows 4% per annum
Independent rates of mortality AM92 Select
Independent rate of withdrawal 10% per annum in the first policy year; 5% per annum in the second andsubsequent policy years.
Initial expenses £150 plus 100% of the amount of initialcommission
Renewal expenses £50 per annum on the second and subsequent premium dates
Initial commission 10% of first premium
Renewal commission 2.5% of the second and subsequent year's premiums
Risk discount rate 8% per annum

(i) Calculate the profit margin on the assumption that the office does not zeroisefuture negative cashflows and that decrements are uniformly distributed overthe year. [13]

(ii) Suppose the office does zeroise future negative cashflows.

(a) Calculate the expected provisions that must be set up at the end of eachyear, per policy in force at the start of each year.

(b) Calculate the profit margin allowing for the cost of setting up theseprovisions. [4]

[Total 17]

End of Paper