NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF COMMERCE

DEPARTMENT OF INSURANCE AND ACTUARIAL SCIENCE

B COMM ACTUARIAL SCIENCE PART IV

ACTUARIAL STATISTICS IV [CIN 4211]

APRIL/MAY 2006 EXAMINATION

TIME ALLOWED: THREE (3) HOURS

INSTRUCTIONS TO CANDIDATES

1. **Answer ALL 11 questions**
2. **Write your student number on the answer booklet**
3. **Mark allocations are shown in brackets**
4. **Begin your answer to each question on a separate sheet**
5. **Credit will be awarded for clarity of answers**
6. **All numerical computations must be clearly shown**

ADDITIONAL MATERIAL

- **An electronic calculator**
- **Actuarial Examination Tables**

1. An insurance company has to estimate the risk premium for the coming year for a certain risk.

State the differences between the assumptions in Empirical Bayes Credibility Theory Model 1 and Model 2, and state why Model 2 is more likely to be useful in practice.

[3 marks]

2. The loss function under a decision problem is given by:

	Θ_1	Θ_2	Θ_3
D_1	23	34	16
D_2	30	19	18
D_3	23	27	20
D_4	32	19	19

(i) State which decision can be discounted immediately and explain why. **[2 marks]**

(ii) Determine the minimax solution to the problem. **[2 marks]**

(iii) Given the following distribution $P(\Theta_1) = 0.25$, $P(\Theta_2) = 0.15$, $P(\Theta_3) = 0.60$, determine the Bayes criterion solution to the problem. **[2 marks]**

[Total 6 marks]

3. Consider a portfolio of insurance policies, on which the number of claims has a binomial distribution with parameters *n* and *p*. The claim size distribution is assumed to be exponential with mean $\frac{1}{\lambda}$. Claims are assumed to be independent random variables and to be independent of the number of claims.

The insurer arranges excess of loss reinsurance, with retention M.
Calculate the moment generating function of S_I, where S_I is aggregate annual claims paid by the insurer (net of reinsurance). **[6 marks]**

4. The total loss to a general insurance office with respect to a particular portfolio over a month is represented by:

$$S = \sum_{i=1}^{N} X_i , \quad N > 0, \qquad (S = 0 \text{ if } N = 0)$$

where N has a Poisson distribution with mean 2 and X_1, X_2, X_3, …… is a sequence of independent and identically distributed random variables that are also independent of N. Their distribution is such that $P(X_i = 1) = P(X_i = 2) = \frac{1}{2}$, where X_i is measured in appropriate monetary units. A reinsurance contract has been arranged such that the amount paid by the reinsurer is S - 3, if S>3 (and 0 otherwise).

Define S_I, S_R as aggregate claims paid by the direct insurer and reinsurer respectively. Calculate $E[S_I]$ and $E[S_R]$. **[8 marks]**

5. In a large portfolio of non-life policies involving a new product, let θ denote the proportion of policies on which claims are made in the first year. The value of θ is unknown and is assumed to have a beta prior distribution with parameters α and β and mean μ_0.

(i) If a random sample of *n* such policies gives rise to *x* claims in the first year, show that the posterior mean of θ is given by

$$\omega_n \mu_0 + (1-\omega_n)\frac{x}{n}$$

expressing the weight ω_n as a function of α, β and *n*. **[3 marks]**

(ii) Two alternative assessments A and B, of the prior probability density function of θ are made as follows:

$$f_A(\theta) = 3(1-\theta)^2 \qquad 0 \le \theta \le 1,$$

$$f_B(\theta) = 4\theta^3 \qquad 0 \le \theta \le 1.$$

81 claims subsequently arise during the year from 1,000 randomly selected policies.

(a) Sketch the two prior densities and comment briefly on the nature of these two sets of prior beliefs.

(b) Determine the posterior Bayes estimate of θ for each prior assessment based on the squared error loss function, and comment briefly on these posterior estimates.

[5 marks]
[Total 8 marks]

6. On 1 January 2001 an insurer in a far off land sells 100 policies, each with a five year term, to householders wishing to insure against damage caused by fireworks. The insurer charges annual premiums of $600 payable continuously over the life of the policy.

The insurer knows that the only likely date a claim will be made is on the day of St Ignitius' feast on 1 August each year, when it is traditional to have an enormous fireworks display. The annual probability of a claim on each policy is 40%. Claim amounts follow a Pareto distribution with parameters α = 10 and λ = 9,000.

(i) Calculate the mean and standard deviation of the annual aggregate claims. **[4 marks]**

(ii) Denote by $\psi(U,t)$ the probability of ruin before time t given initial surplus U.

(a) Explain why for this portfolio $\psi(U,t_1) = \psi(U,t_2)$ if $\frac{7}{12} < t_1, t_2 < \frac{19}{12}$. **[1 mark]**

(b) Estimate $\psi(15{,}000, 1)$ assuming annual claims are approximately normally distributed. **[4 marks]**

[Total 9 marks]

7. The loss amount, X, on a certain type of insurance policy, has a Pareto distribution with density function *f(x)*, where:

$$f(x) = \frac{3 \times 400^3}{(400+x)^4} \qquad (x > 0)$$

A policyholder deductible of $100 is applied to this policy.

(i) Calculate the expected claim size paid by the insurance company. **[5 marks]**

(ii) Comment on the difference between your answer to (i) and the expected loss amount, E[X]. **[2 marks]**

[Total 7 marks]

8. Cumulative claims incurred on a motor insurance account are as follows:

Figures in $000's

	Development year			
Policy year	0	1	2	3
2000	1,417	1,923	2,101	2,209
2001	1,701	2,140	2,840	
2002	1,582	1,740		
2003	2,014			

The data have already been adjusted for inflation. Annual premiums written in 2003 were $3,912,000 and the ultimate loss ratio has been estimated as 92%. Claims paid to date for policy year 2003 are $561,000, and claims are assumed to be fully run-off by the end of development year 3.

Estimate the outstanding claims to be paid arising from policies written in 2003 **only**, using the Bornhuetter-Ferguson technique. **[9 marks]**

9. The no claim discount system for a particular class of annual insurance policy has three categories with discount levels of 0%, 30% and 50%. If a policyholder makes any claims during the year he or she moves down a single category (or stays at the 0% discount level). If no claims are made, then the policyholder moves to the next higher category (or stays at the 50% discount level).

The probability that a policyholder will make at least one claim in any one year is:

p in the 0% discount category
$0.8p$ in the 30% discount category
$0.6p$ in the 50% discount category

The premium charged at the 0% discount category is c.

(i) Write down the transition matrix for this system in terms of p. **[2 marks]**

(ii) Derive the steady state distribution of policyholders in each discount category in terms of p. **[5 marks]**

(iii) Calculate the average premium paid, $A(p, c)$, in the steady state in terms of p and c.

[3 marks]
[Total 10 marks]

10. A generalised linear model (GLM) has independent Poisson responses $\{Y_{ix}\}$, with

$$E(Y_{ix}) = m_{ix}, \qquad Var(Y_{ix}) = m_{ix}.$$

The linear parameterised predictor η_{ix} is linked to the mean response by the log function, such that

$$\log\ m_{ix} = \eta_{ix}.$$

(i) (a) Write down an expression for the variance function V(.) for this GLM.

(b) Evaluate the integral expression

$$d_{ix} = 2\int_{\hat{m}_{ix}}^{y_{ix}} \frac{y_{ix} - t}{V(t)}\, dt$$

to determine an expression for the general component d_{ix} of the deviance of this GLM in terms of the observed responses y_{ix} and fitted values $\hat{m}_{ix}$.

(c) Write down expressions for the deviance residual and Pearson residual for this model.

[5 marks]

(ii) The actual deaths a_{ix}, with matching exposures r_{ix} to the risk of death, based on policy counts for UK female assured lives in 1987-90, are modelled as the independent responses A_{ix} of a Poisson GLM with

$$E(A_{ix}) = m_{ix} = r_{ix}\mu_{ix}$$

where μ_{ix} is the targeted unknown force of mortality, and

$$\log\ m_{ix} = \eta_{ix} = \log r_{ix} + \log \mu_{ix}.$$

The structure of the linear predictor is generated by the two covariates:

policy duration, denoted by the factor D(*i*), *i* = 1, 2, 3, which represent the three levels 0, 1, or 2+ years respectively; and

age, denoted by the variable *x*, coded in 5 yearly bands, *x* = (mid age band – 17.5)/5.

The term log r_{ix} is a known offset and log μ_{ix} takes various linear parameterised forms as implied by the following computer output:

Fit 1:
scaled deviance = 16829, residual df = 41

Add in straight-line age effects:
scaled deviance = 338.83 (change = 16400),
residual df = 40 (change = 1)

Add in quadratic age effects:
scaled deviance = 306.71 (change = 32.12),
residual df = 39 (change = 1)

Add in cubic age effects:
scaled deviance = 306.51 (change = 0.1976),
residual df = 38 (change = 1)

Add in duration effects:
scaled deviance = 46.055 (change = 260.5),
residual df = 36 (change = 2)

estimate	*s.e.*	*parameter*
-9.504	0.1382	1
0.3370	0.05627	x
0.01011	0.007445	x^2
-0.0001376	0.0003055	x^3
0.3663	0.06365	D(2)
0.6682	0.04979	D(3)

(a) List the various fitted parameterised formulae.

(b) Interpret the output and state your conclusions.

(c) What additional relevant computer output would be useful?

[9 marks]
[Total 14 marks]

11. (i) A random variable X has the lognormal distribution with density function *f(x)* and parameters μ and σ. Show that for $a > 0$

$$\int_a^\infty xf(x)dx = \exp\left(\mu + \frac{\sigma^2}{2}\right)\left(1 - \Phi\left(\frac{\log a - \mu - \sigma^2}{\sigma}\right)\right)$$

where Φ is the cumulative distribution function of the standard normal distribution.

[5 marks]

(ii) Claims under a particular class of insurance follow a lognormal distribution with mean 9.070 and standard deviation of 10.132 (figures in $000s). In any one year 20% of policies are expected to give rise to a claim.

An insurance company has 200 policies on its books and wishes to take out individual excess of loss reinsurance to cover all the policies in the portfolio. The reinsurer has quoted premiums for two levels of reinsurance as follows (figures in $000s):

Retention Limit	*Premium*
25	48.5
30	38.2

(a) Calculate the probability, under each reinsurance arrangement, that a claim arising will involve the reinsurer.

(b) By investigating the average amount of each claim ceded to the reinsurer, calculate which of the retention levels gives the best value for money (ignoring the insurer's attitude to risk).

(c) The following year, assuming all other things equal, the insurer believes that inflation will increase the mean and standard deviation of the claims in its portfolio by 8%. If the reinsurer charges the same premiums as before, which of the retention levels will be best value for money next year?

[15 marks]
[Total 20 marks]

********************** END OF EXAMINATION **********************