NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF COMMERCE

DEPARTMENT OF INSURANCE AND ACTUARIAL SCIENCE

B COMM ACTUARIAL SCIENCE PART IV

ACTUARIAL STATISTICS IV [CIN 4211]

JULY 2006 SUPPLEMENTARY EXAMINATION

TIME ALLOWED: THREE (3) HOURS

INSTRUCTIONS TO CANDIDATES

1. **Answer ALL 11 questions**
2. **Write your student number on the answer booklet**
3. **Mark allocations are shown in brackets**
4. **Begin your answer to each question on a separate sheet**
5. **Credit will be awarded for clarity of answers**
6. **All numerical computations must be clearly shown**

ADDITIONAL MATERIAL

- **An electronic calculator**
- **Actuarial Examination Tables**

1. The random variable Y is defined as:

$$Y = \sum_{i=1}^{N} X_i$$

where the X_i's are independent random variables, each with an exponential distribution with mean ϕ, and N is a non-negative integer-valued random variable, independent of the X_i's, with probability generating function:

$$G(t) = p/(1-qt)$$

where $0 < p < 1$ and $q = 1 - p$.

Find the moment generating function of Y. **[3 marks]**

2. Claims occur on a portfolio of insurance policies according to a Poisson process at a rate λ. All claims are for a fixed amount *d*, and premiums are received continuously. The insurer's initial surplus is U ($< d$) and the annual premium income is $1.2\lambda d$.
Show that the probability that ruin occurs at the first claim is:

$$1 - e^{-\frac{1}{1.2}(1-\frac{U}{d})}.$$ **[4 marks]**

3. An insurance company has an excess of loss reinsurance contract, with retention \$50,000. Over the last year, the insurer paid the following claims (in \$'s):

12,372	2,621	389	43,299

In addition, the insurer paid the retained amount of \$50,000 on 6 claims with the excess being paid by the reinsurer.
The insurer believes that the distribution of gross claim amounts is exponential, with mean μ.

Calculate the maximum likelihood estimate of μ based on the information given above.
[5 marks]

4. An actuarial student is considering his post-examination holiday plans. His travel agent tells him that there will definitely be vacancies in either Hotel A or Hotel B but not at both. He must book his flight now to take advantage of current huge discounts on flight prices. Flights to airport 1 cost \$110 and flights to airport 2 cost \$160. Accommodation at Hotel A costs \$890 and accommodation at Hotel B costs \$490. If he chooses to fly to airport 1, a taxi to Hotel A will cost \$10 and a taxi to Hotel B will cost \$70. If he chooses to fly to airport 2, a taxi to Hotel A will cost \$65 and a taxi to Hotel B will cost \$5. The student believes there is a 90% chance of the vacancies being at Hotel B. To which airport should he book a flight in order to minimise the expected cost? **[4 marks]**

5. A risk consists of 5 policies. On each policy in one month there is exactly one claim with probability θ and there is negligible probability of more than one claim in one month. The prior distribution for θ is uniform on (0,1). There are a total of 10 claims on this risk over a 12 month period.

(i) Derive the posterior distribution for θ. **[2 marks]**

(ii) Determine the Bayesian estimate of θ under:

(a) quadratic loss
(b) all-or-nothing loss

[3 marks]
[Total 5 marks]

6. A generalised linear model has independent Binomial responses $Z_1, \ldots\ldots\ldots\ldots, Z_k$ with $E(Z_i) = n\mu, \quad Var(Z_i) = n\mu(1-\mu)$ for $0 < \mu < 1$.

(i) Show that $Y_i = Z_i / n$ belongs to an exponential family. **[2 marks]**

(ii) Identify the natural parameter and the canonical link function, and derive the variance function. **[4 marks]**
[Total 6 marks]

7. The following table shows incremental claims relating to the accident years 2003, 2004 and 2005. It is assumed that claims are fully run-off by the end of development year 2. Estimate total outstanding claims using the chain-ladder technique, ignoring inflation.

Incremental Claims

Accident Year	*Development Year* 0	1	2
2003	2587	1091	251
2004	2053	1298	
2005	3190		

[7 marks]

8. $\{S_n\}_{n=1}^{\infty}$ is a sequence of independent and identically distributed random variables, each with mean 5 and variance 25. S_n represents the aggregate claims from a risk in year *n*. The insurer intends to calculate the annual risk premium, Π, for this risk so that:

$$\Pr[S_n > \Pi] = 0.01$$

(i) Assuming S_n has an exponential distribution, show that $\Pi = 23.03$. **[2 marks]**

(ii) Calculate the value of Π assuming S_n has a lognormal distribution. **[6 marks]**

(iii) Comment on the difference between your answers to parts (i) and (ii).
[2 marks]

(iv) Assuming that S_n has an exponential distribution, calculate the value of:

$$P[(S_1 \le 23.03) \text{ and } (S_1 + S_2 \le 46.06)]$$ **[5 marks]**

[Total 15 marks]

9. An insurance company charges an annual premium of \$300 and operates a No Claims Discount system as follows:

Level 1	0% discount
Level 2	30% discount
Level 3	60% discount

The rules for moving between levels are as follows:

If the policyholder does not make a claim during the year, they move up one level or are eligible to stay at level 3.

If the policyholder makes 1 claim during the year, they move down one level or stay at level 1.

If the policyholder makes 2 or more claims during the year, they move straight down to, or remain at, level 1.

The insurance company has recently introduced a "protection" system where on reaching, or remaining eligible to remain at, level 3, policyholders are immediately offered the opportunity to "protect" their discount for an additional annual premium of \$50. If they make no claims or 1 claim during the year they can remain at level 3. However, if they make 2 or more claims during the year, they move straight down to level 1.

Out of the policyholders who had "protected" their discount level at the beginning of the year and are still eligible to stay at level 3 at the end of the year, 25% choose to "protect" their discount again the following year. From all the other policyholders eligible for level 3 at the end of the year, 10% choose to take the "protection" option.

Policyholders at different levels are found to experience different rates of claiming. The number of claims made per year follows a Poisson distribution with parameter λ as follows:

Level:	*1 and 2*	*3 and "protected"*
λ:	0.40	0.25

(i) Derive the transition matrix. **[6 marks]**

(ii) Calculate the proportions at each of the levels 1, 2, 3 and the "protected" level, when the system reaches a steady state. **[8 marks]**

(iii) Determine the average premium per policy. **[2 marks]**

[Total 16 marks]

10. (i) In the context of Empirical Bayes Credibility Model 2, and with the usual notation:

(a) give a brief intuitive explanation of what $E[s^2(\theta)]$ and $V[m(\theta)]$ represent, and

(b) hence, or otherwise, state which of the following two formulae is correct for the credibility factor, Z, giving brief reasons for your answer

$$Z = \sum_{j=1}^{n} P_j \Big/ \left\{ \sum_{j=1}^{n} P_j + E[s^2(\theta)]/V[m(\theta)] \right\}$$

or,
$$Z = \sum_{j=1}^{n} P_j \Big/ \left\{ \sum_{j=1}^{n} P_j + V[m(\theta)]/E[s^2(\theta)] \right\}$$
[6 marks]

(ii) For the past five years an insurance company has insured 15 different chains of newsagents' shops against damage to their premises and stock from any cause. For chain *i*, *i* = 1, 2, …….., 15, and year *j*, *j* = 1, 2, ……, 5, the random variable Y_{ij} represents the annual claims and P_{ij} represents the number of shops in the chain. The sequence $\left\{\{Y_{ij}; P_{ij}\}_{j=1}^{5}\right\}_{i=1}^{15}$ satisfies all the assumptions for Empirical Bayes Credibility Theory Model 2. The data for the first three chains in this collection are shown in the table below. Also shown for each of the first two chains is the credibility premium per shop for the coming year.

	$Y_{ij}; P_{ij}$					
Chain	*j* = 1	2	3	4	5	*Credibility premium per shop*
1	450; 2	220; 2	3700; 2	250; 2	380; 2	750
2	2500; 3	1140; 4	3600; 4	3900; 4	860; 5	733
3	4950; 9	39600; 9	14850; 11	29700; 12	9900; 14	

(a) Calculate the credibility premium per shop for the coming year for chain number 3.

(b) Explain carefully why the credibility premium per shop for the coming year is higher for chain 1 than for chain 2 even though the average annual claim per shop is lower for chain 1 than for chain 2.

[11 marks]
[Total 17 marks]

11. (i) The random variable X has the lognormal distribution with density function *f(x)* and parameters μ and σ. Show that for any real number *a* > 0 and any positive integer *k*

$$\int_0^a x^k f(x)dx = \exp\left(k\mu + \frac{k^2\sigma^2}{2}\right)\Phi\left(\frac{\log a - \mu - k\sigma^2}{\sigma}\right)$$

where Φ is the cumulative distribution function of the standard normal random variable.

[6 marks]

(ii) The amount, X, of a claim , in thousands of dollars, from an insurance portfolio has the lognormal distribution with mean 12.2 and standard deviation 16. Consider an excess of loss reinsurance policy with a policy retention of $28,000 so that the claim paid by the insurer ($'000) is given by X_I, where

$$X_I = \begin{cases} X & X \leq 28 \\ 28 & X > 28. \end{cases}$$

(a) Determine the probability that a claim involves the reinsurer. **[2 marks]**

(b) Calculate the mean and variance of the claims paid by the insurer.

[6 marks]

(c) Given that a claim is referred to the reinsurer, what is the conditional expected value paid by the reinsurer.

[4 marks]

[Total 18 marks]

***** END OF EXAMINATION *****