NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF COMMERCE

DEPARTMENT OF INSURANCE AND ACTUARIAL SCIENCE

B.COMM (HONOURS) DEGREE IN ACTUARIAL SCIENCE

CIN 4211 ACTUARIAL STATISTICS IV

AUGUST 2009-SECOND SEMESTER EXAMINATIONS

DURATION: 3 HOURS

Instructions to Candidates

1. **Attempt all Questions**
2. **To Obtain Full Marks Show ALL appropriate steps to your answers**

Requirements

1. **Actuarial Tables**
2. **Non-programmable Scientific calculator**

Question 1

i) State 2 conditions for a risk to be insurable. [2 marks]

ii) Describe briefly the five distinct types of liability insurance [5 marks]

Total 7 marks

Question 2

The table below shows the cumulative incurred claims by year of accident and earned premiums for a particular type of household property. Claims paid to date amount to $14,500.

Development Year

Accident Year	**0**	**1**	**2**	**3**	**Earned Premiums**
2005	3,340	3,750	4,270	4,400	4,800
2006	3,670	4,080	4,590	-	4,900
2007	3,690	4,290	-	-	5,050
2008	4,150	-	-	-	5,200

Use the Bornhuetter-Ferguson method to calculate the reserve for outstanding claims, assuming that the claims are fully developed by the end of development year 3. You may ignore the effect of inflation. [13 marks]

Total 13marks

Question 3

The loss function $\theta_i\left(i=1,2,3\right)$ under a decision problem is given under the table below;

	θ_1	θ_2	θ_3
D_1	22	18	38
D_2	20	26	34
D_3	14	26	20
D_4	32	10	26

i) State which of the 4 decisions can be discounted immediately and explain why. [3 marks]

ii) Explain what is meant by the minimax criteria and further determine the minimax solution. [5 marks]

Total 8 marks

Question 4

Derive the auto covariance and auto correlation of the
$AR(1)$ *process* $X_t = \beta X_{t-1} + e_t$

where $|\beta| < 1$ and the e_t forms a white noise process [5 marks]

Total 5 marks

Question 5

i) Explain the disadvantages of using truly random variables, as opposed to pseudo random numbers. [3 marks]

ii) List 4 distinct methods for the generation of random variables [4 marks]

Total 7marks

Question 6

Suppose claim amounts, X, on an insurance portfolio, follow an exponential distribution with mean \$200. A reinsurance policy is arranged such that the re-insurer pays
X_R *where*:

$$X_R = \begin{cases} 0 \;\; if X \le \$50 \\ X - 50 \;\; if \;\; \$50 < X \le S \\ S - 50 \;\; if \;\; X > 5 \end{cases}$$

Calculate S such that $E[X_R] = \$100$ [10 marks]

Total 10 marks

Question 7

The aggregate claims arising during each year from a particular policy are assumed to follow a normal distribution with mean 0.7P and standard deviation 2P, where P is the earned premium. An insurer with an initial surplus of $100,000 expects to sell 100 policies at the beginning of the coming year in respect of risks for an annual premium of $5,000. The insurer will incur expenses of 0.2P at the time of writing each policy.

i) Show that the surplus at the end of the year is $\Psi(_1) = 500{,}000 - S(1)$;

Where $S(1)$ = Claims incurred during the year. [3 marks]

ii) Calculate the probability that the insurer will be insolvent at the end of the year in respect of this portfolio only. [7 marks]

Total 10 marks

Question 8

The no claim discount (NCD) system operated by an Insurance Company has 3 levels of discount 0%, 25% and 50%. If a policyholder makes a claim, they remain at or move down to the 0% discount level for two years. Otherwise they move up a discount level in the following year or remain at the maximum 50% level. The probability of an accident depends on the discount level as follows:

Discount Level	**Probability of Accident**
0%	0.25
25%	0.20
50%	0.10

The full premium payable at the 0% discount level is 750. Losses are assumed to follow a lognormal distribution with mean 1,451 and standard deviation 604.4. Policyholders will only claim if the loss is greater than the total additional premiums that would have to be paid over the next three years, assuming that no further accidents occur.

i) Calculate the smallest loss for which a claim will be made for each of the four states in the NCD system. [3 marks]

ii) Derive the transition matrix for this NCD system [7 marks]

iii) Calculate the proportion of policyholders at each discount level when the system reaches a stable state. [4 marks]

iv) Determine the average premium paid once the system reaches a stable state. [3 marks]

v) Describe the limitations of simple NCD systems such as this one [3 marks]

Total 20 marks

Question 9

i) Explain the concept of co-integrated time series [3 marks]

ii) Give two examples of circumstances when it is reasonable to expect that two processes may be co-integrated. [2 marks]

Total 5marks

.Question 10

State the Markov property and explain briefly whether the following processes are Markov.
AR (4);
ARMA $(1,\ 1)$ [5 marks]

Total 5marks

Question 11

A portfolio of general insurance policies is made up of two types of policies. The policies are assumed to be independent and claims are assumed to occur according to a Poisson process. The claim severities are assumed to have exponential distributions. For the first type of policy, a total of 20 claims are expected each year and the expected size of a claim is $4,500.

i) Calculate the mean and variance of the total cost of annual claims, S, arising from this portfolio. The risk premium loading is denoted by θ, so that the annual premium on each policy is
$(1+\theta)\gamma$ expected annual claims on each policy.
The initial reserve is denoted by U.

A normal approximation is used for the distribution of S, and the initial reserve is set by ensuring that $p(S < U + annual \ \ premium) = 0.975$.

ii) a) Derive an equation for U in terms of θ
b) Determine the annual premium required in order that no initial reserve is necessary [7 marks]

Total 10 marks

End of Examination