NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF COMMERCE

DEPARTMENT OF INSURANCE AND ACTUARIAL SCIENCE

CIN 4211- ACTURIAL STATISTICS PROBABILITY THEORY IV

MAY/JUNE 2011 –FINAL EXAMINATIONS

DURATION: 3 HOURS

Instructions to candidates

1. Attempt all 12 Questions
2. To Obtain Full Marks Show ALL appropriate steps to your answers

Requirements

1. Actuarial Tables (2002) Edition
2. Non- programmable scientific calculator

QUESTION 1

Claims occur on a portfolio of general insurance policies independently and at random, Claims are classified as "type A claims" or type B claim". It is estimated that 10% of all claims are type B claims.

Type A claims are distributed according to Pareto distribution with parameters α=3.5 and λ =200. Type B claims are distributed according to a Pareto distribution with parameters α = 4 and λ =1200.

Let X denote a claim chosen at random from this portfolio.

(a) Calculate Pr (X>2000) [3 marks]

(b) Calculate E [X] and Var [X] [4 marks]

Total 7 marks

QUESTION 2

The table below gives the cumulative incurred claims by year of accident and development year, and the earned premium (EP) income by accident year. It can be assumed that the data have already been adjusted for inflation.

Accident Year	Incurred claims by the end of development year and earned premium income					
	0	1	2	3	4	EP
1	3600	4230	4560	4802	5000	6000
2	4180	4685	5333	5498		6200
3	4586	5100	5741			6300
4	4615	5357				6450
5	5191					6600

Use the Borhuetter-Ferguson method to estimate the total reserve required to meet the outstanding claims. You may assume that the total amount of claims paid to date is $25 000. You may also assume that you can estimate the expected loss ratio as that experienced for the fully developed first accident year.

[13 marks]

Total 13 marks

QUESTION 3

Let S denote the aggregate claims in a single year form a risk. We assume that:

$S \sim N(\mu,\sigma^2)$ where μ is unknown but σ is known. The correct premium to be charged each year for this risk, before adding a profit loading is μ.

Suppose our prior information/belief about the fixed value of μ is summarized in the following prior distribution

$\mu \sim N(\mu_0, \sigma_0^2)$

Where both σ_0 and μ_0 are known values.

i) Without any data from this risk, what is your estimate of the value of μ? [3 marks]

ii) Now suppose you have observed the risk for n years and the aggregate claims in successive years are $X_1, X_2, \ldots, X_n$. Show that the Bayesian estimator of μ (with respect to squared error loss) is in the form of a credibility estimate. [10 marks]

Total 13 marks

QUESTION 4

If X has the Burr distribution, then

$$P_r(X \leq X) = 1 - \left(\frac{\lambda}{\lambda + x^{\gamma}}\right)^{\alpha}, x \rangle 0$$

Let U = U (0, 1) . Use the probability integral transform to generate $X \sim \beta(\alpha,\lambda,\gamma)$

[7 marks]

Total 7 marks

QUESTION 5

Suppose that the random variable Y has a Poisson distribution with probability function:

$$f(y;\lambda) = \exp(-\lambda)\frac{\lambda^{y}}{y!}$$

i) Show that Y is a member of the exponential family. [5 marks]

ii) Identify the natural parameter θ, the scale parameter ϕ, and the functions *a (.), b (.) and c (., .)* [5 marks]

Total 10 Marks

QUESTION 6

(a) Consider the MA(2), AR(2) and ARMA (2, 2) models

$$X_t = \psi_1\varepsilon_{t-1} + \psi_2\varepsilon_{t-2} + \varepsilon_t$$
$$X_t = \phi X_{t-1} + \phi_2 X_{t-2} + \varepsilon_t$$
$$X_t = \phi_1 X_{t-1} + \varphi_2 X_{t_{-2}} + \psi_1\varepsilon_{t-1} + \psi_2 E_{t-2} + \varepsilon_t$$

Where ε_t are iid innovations with $E(\varepsilon_t) = 0$ and variance $(\varepsilon_t) = \sigma_\varepsilon^2$. Give the necessary and sufficient conditions on the coefficients so that:

i) The MA(2) model is invertible. [4 marks]

ii) The AR(2) model is causal and stationary [4 marks]

iii) T he ARMA(2,2) model is causal, stationary and invertible. [2 marks]

(b) Given a random walk defined by:

$$X_t = \begin{cases} 0 & ,t=0 \\ X_{t-1}+\varepsilon_t & ,t>0 \end{cases}$$

Where ε_t are iid with mean zero and variance σ_ε^2. Calculate $E(X_t)$ and $V(X_t)$ for t>0. Show that X_t is non-stationary. [6 marks]

(c) Given an MA(3) model

$X_t=0.3\varepsilon_{t-1} +0.4\varepsilon_{t-2} + 0.2\varepsilon_{t-3} +\varepsilon_t$, where ε_t are iid with $E(\varepsilon_t) = 0$ and VAR $(\varepsilon_t)= \sigma_\varepsilon^2$

i) Calculate γ(k) for this model, $k = 0,\pm1,\pm2,\ldots\ldots\ldots\ldots$ [6 marks]

ii) Show that this MA(3) model is invertible by means of some simple conditions. [3 marks]

Total 25 Marks

QUESTION 7

a) By minimizing

$$E\{(E[m(\theta)\underline{X}] - a - b\overline{X})^2\}$$

Show that :

i) $a = (1-b)E[m(\theta)]$ [4 marks]

ii) $b = \dfrac{n}{n + \dfrac{E[s^2(\theta)]}{V[m(\theta)]}}$ [8 marks]

b) An insurer has 5 industrial fire insurance policies in its portfolio. Observed values of the aggregate claims for these 5 risks over 6 years are shown in the table below (in units of $1000). Use EBCT 1 to calculate the premium next year (before allowing for profit and expenses) for each of these 5 risks.

		Year					
		1	2	3	4	5	6
Risk	1	104	75	34	104	80	89
	2	114	139	31	95	106	73
	3	137	157	123	125	79	141
	4	93	108	110	112	118	83
	5	68	134	66	94	119	89

[13 marks]

Total 25 marks

*******************END OF EXAMINATION*********************