NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF INDUSTRIAL TECHNOLOGY

DEPARTMENT OF CIVIL AND WATER ENGINEERING

STRUCTURAL ANALYSIS I

TCW 3102

Examination Paper

December 2016

This examination paper consists of 8 pages

Time Allowed: 3 hours

Total Marks: 100

Special Requirements: None

Examiner's Name: Miss Diana Makweche/ Mrs Faith Makwiranzou

INSTRUCTIONS

1. Answer any four (4) questions. Credit will not be given for additional questions attempted.
2. Each question carries 25 marks.
3. Where relevant, use the solution method prescribed.

MARK ALLOCATION

QUESTION	MARKS
1.	**25**
2.	**25**
3.	**25**
4.	**25**
5.	**25**
TOTAL	**100**

QUESTION 1

(i) Analyse the beam in Figure Q1A for reactions, bending moment diagram and shear force diagram. [6]

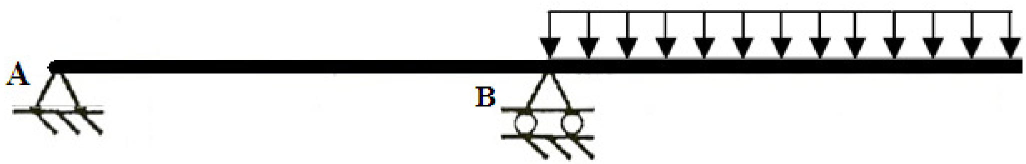


Figure Q1A

(ii) Figure Q1B shows two frames. Sketch the deflected shape of each and explain the differences observed. [6]

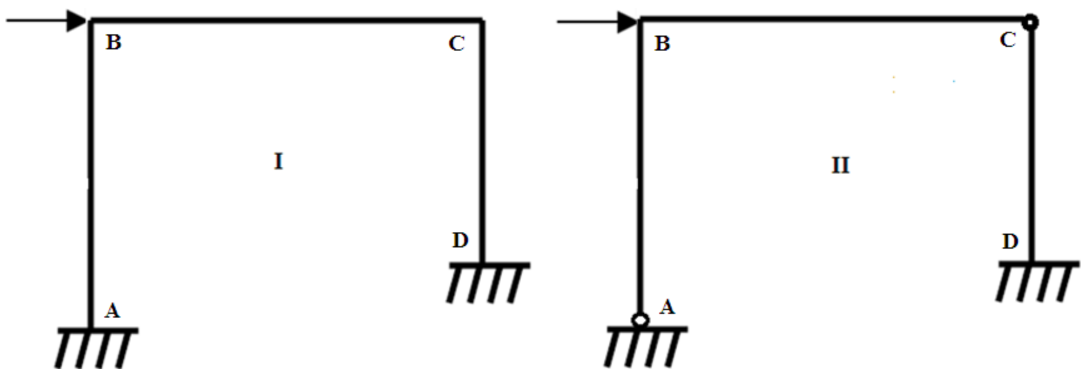


Figure Q1B

(iii) Construct the influence line for truss member CH in the truss of Figure Q1C. (Use the Method of Sections to obtain the truss member forces). [10]

(iv) What is the maximum compressive force in the member due to a uniformly distributed load of intensity 10kN/m? [3]

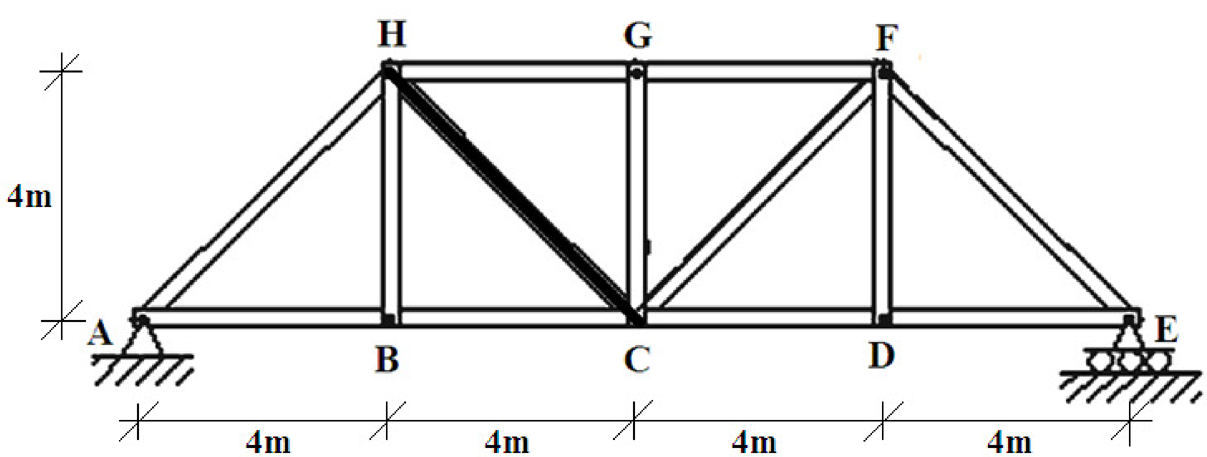


Figure Q1C

QUESTION 2

The propped cantilever in Figure Q2A experiences a settlement of 12mm at support B under the action of a uniformly distributed load of $15kN/m$.

(i) Using the Flexibility Method, determine the reactions. [15]
Take $E = 200GPa\ (200x10^6 kN/m^2)$ and $I = 80x10^6\ mm^4$.

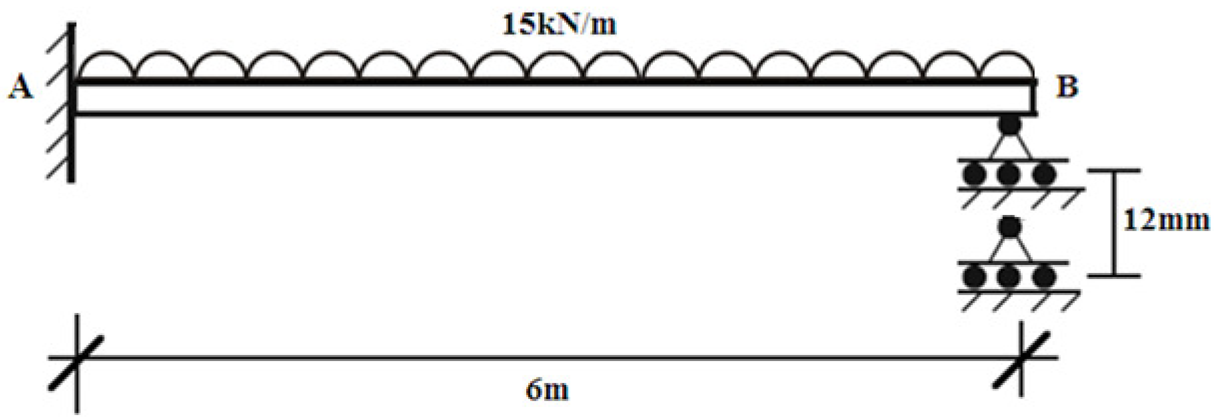


Figure Q2A

(ii) A statically indeterminate truss is to be analysed. Using the information provided in Figure Q2B, construct a table and calculate the truss member forces. $A = 200mm^2$and $E = 200GPa\ (200x10^6 kN/m^2)$. [10]

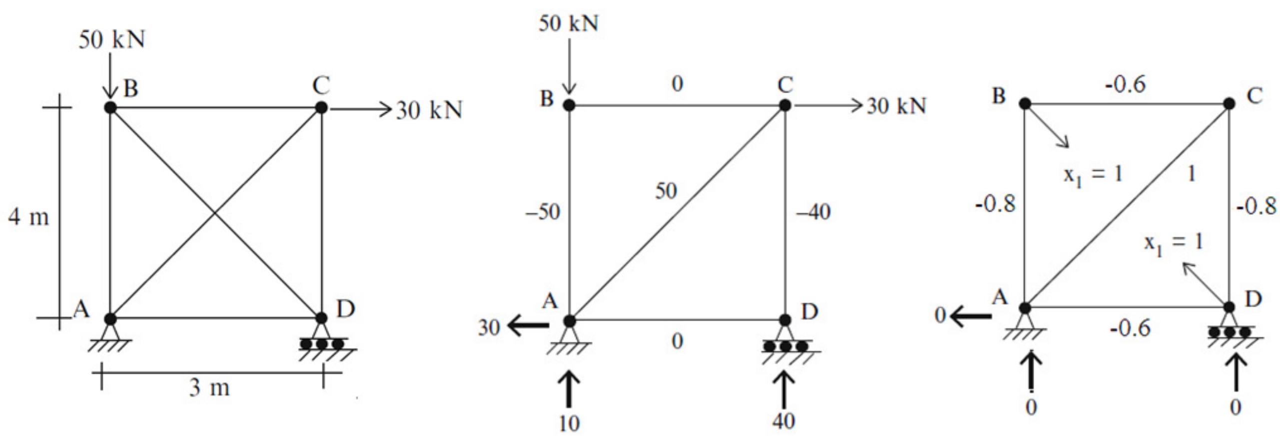


Figure Q2B

QUESTION 3

Figure Q3 shows a frame of constant flexural rigidity (EI).

(i) Sketch the deflected shape, and clearly indicate the angles through which the joints rotate. [3]

(ii) Find the magnitude of the fixed end moments. [2]
(iii) Determine the member end moments using the Slope Deflection Method. [10]
(iv) Calculate the reactions. [4]
(v) Construct the bending moment and shear force diagrams. [6]

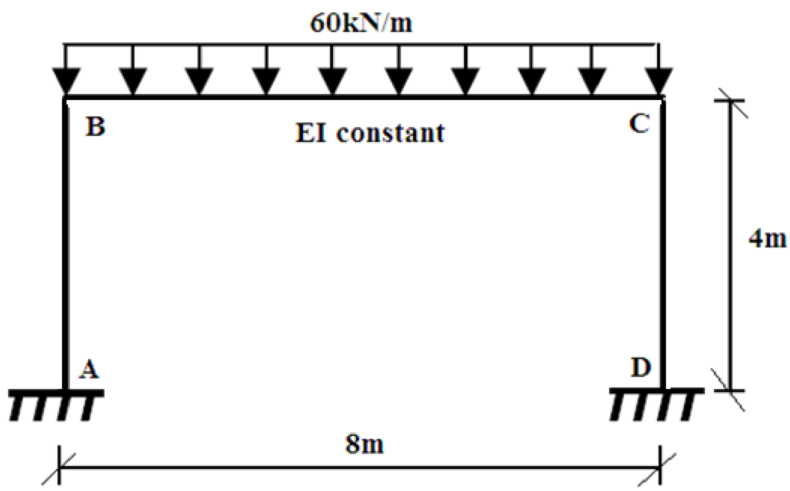


Figure Q3

QUESTION 4

The beam of Figure Q4 is fully fixed at the end supports A and D; and continuous over internal supports B and C. $E = 200GPa\ (200x10^6 kN/m^2)$.

(i) Calculate the stiffness factors. [3]
(ii) Determine the distribution factors. [3]
(iii) Find the fixed end moments. [2]
(iv) Use the Moment Distribution Method to determine the end moments in each span. [7]
(v) Determine the reactions. [4]
(vi) Draw the bending moment and shear force diagrams. [6]

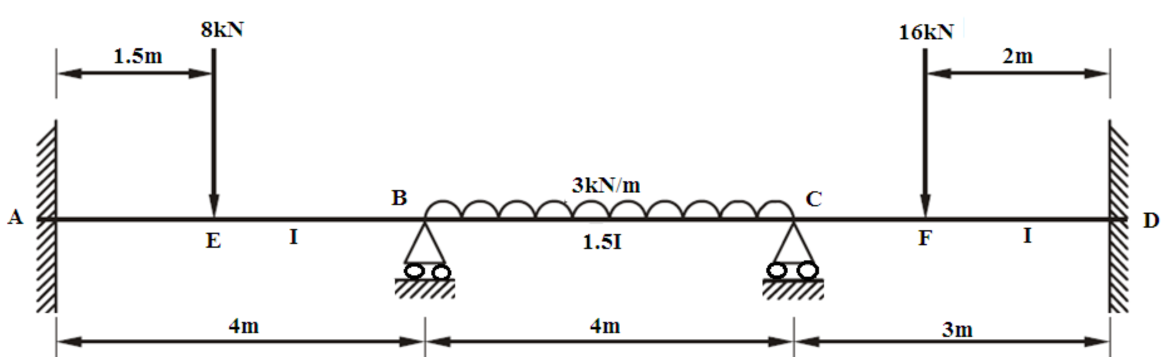


Figure Q4

QUESTION 5

Figure Q5 shows a continuous beam that is fixed at one end, A, and sits on rollers at B and C. Using two beam elements of equal length

(i)	Determine the equivalent nodal forces at the end of each element.	[4]
(ii)	Construct the stiffness matrix for each element.	[4]
(iii)	Assemble the global stiffness matrix.	[5]
(iv)	Reduce the global system of equations by applying the boundary conditions.	[4]
(v)	Calculate the unknown displacements and the forces at the ends of the elements and, hence, the reactions.	[8]

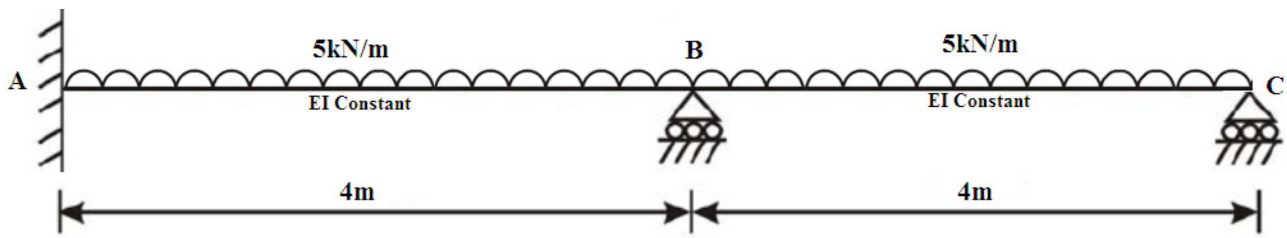
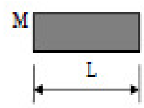
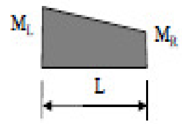
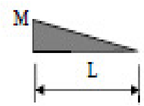
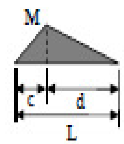
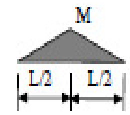
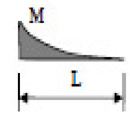
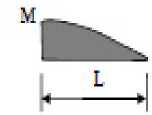
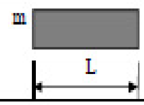
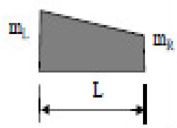
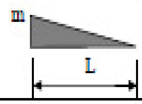
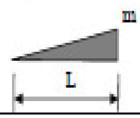
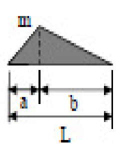
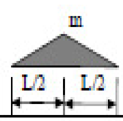


Figure Q5

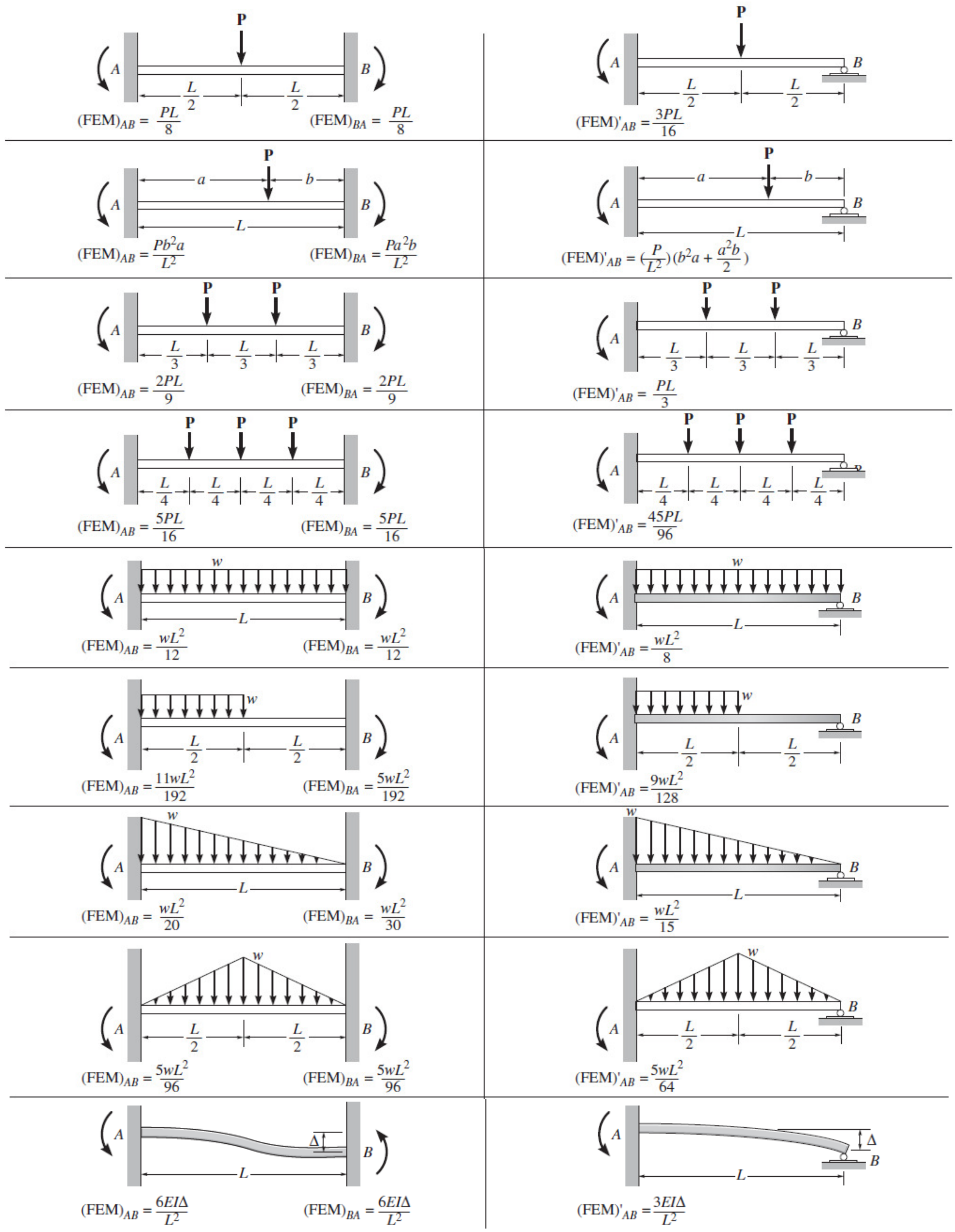
Integral Table

To Evaluate Product Integrals of the Form: $\int_0^L mM\,dx$

							
	LmM	$\frac{L}{2}m(M_L+M_R)$	$\frac{L}{2}mM$	$\frac{L}{2}mM$	$\frac{L}{2}mM$	$\frac{L}{3}mM$	$\frac{2L}{3}mM$
	$\frac{L}{2}(m_L+m_R)M$	$\frac{L}{6}[m_L(2M_L+M_R)$ $+m_R(M_L+2M_R)]$	$\frac{L}{6}(2m_L+m_R)M$	$\frac{L}{6}\left[m_L\left(1+\frac{d}{L}\right)\right.$ $\left.+m_R\left(1+\frac{c}{L}\right)\right]M$	$\frac{L}{4}(m_L+m_R)M$	$\frac{L}{12}(3m_L+m_R)M$	$\frac{L}{12}(5m_L$ $+3m_R)M$
	$\frac{L}{2}mM$	$\frac{L}{6}m(2M_L+M_R)$	$\frac{L}{3}mM$	$\frac{L}{6}m\left(1+\frac{d}{L}\right)M$	$\frac{L}{4}mM$	$\frac{L}{4}mM$	$\frac{5L}{12}mM$
	$\frac{L}{2}mM$	$\frac{L}{6}m(M_L+2M_R)$	$\frac{L}{6}mM$	$\frac{L}{6}m\left(1+\frac{c}{L}\right)M$	$\frac{L}{4}mM$	$\frac{L}{12}mM$	$\frac{L}{4}mM$
	$\frac{L}{2}mM$	$\frac{L}{6}m\left[M_L\left(1+\frac{b}{L}\right)\right.$ $\left.+M_R\left(1+\frac{a}{L}\right)\right]$	$\frac{L}{6}m\left(1+\frac{b}{L}\right)M$	$\frac{(L^2-a^2-c^2)}{6bc}LmM$ only for $a < c$	$\left(\frac{3L^2-4a^2}{12bL}\right)LmM$	$\frac{L}{12}m\left(1+\frac{b}{L}\right.$ $\left.+\frac{b^2}{L^2}\right)M$	$\frac{L}{12}m\left(5-\frac{a}{L}\right.$ $\left.-\frac{a^2}{L^2}\right)M$
	$\frac{L}{2}mM$	$\frac{L}{4}m(M_L+M_R)$	$\frac{L}{4}mM$	$\left(\frac{3L^2-4c^2}{12dL}\right)LmM$	$\frac{L}{3}mM$	$\frac{7L}{48}mM$	$\frac{17L}{48}mM$

(Caprani C- Lecture Notes)

Fixed End Moments



Slope Deflection Equations

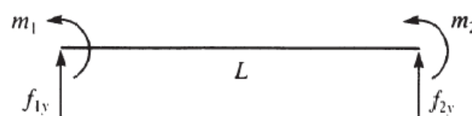
$$M_N = 2Ek(2\theta_N + \theta_F - 3\psi) + (FEM)_N$$

$$M_N = 3Ek(\theta_N - \psi) + (FEM)_N$$

Beam Stiffness Matrix

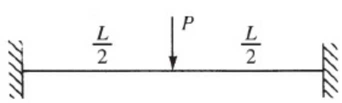
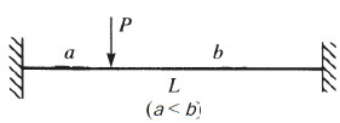
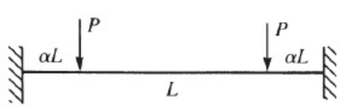
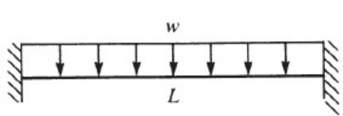
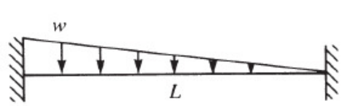
$$\begin{Bmatrix} f_{1y} \\ m_1 \\ f_{2y} \\ m_2 \end{Bmatrix} = \frac{EI}{L^3} \overset{\begin{matrix} v_1 & \quad \phi_1 & \quad v_2 & \quad \phi_2 \end{matrix}}{\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}} \begin{Bmatrix} v_1 \\ \phi_1 \\ v_2 \\ \phi_2 \end{Bmatrix}$$

Equivalent Nodal Forces



Positive nodal force conventions

Equivalent Nodal Forces for Different Load Types

f_{1y}	m_1	Loading case	f_{2y}	m_2
$\frac{-P}{2}$	$\frac{-PL}{8}$		$\frac{-P}{2}$	$\frac{PL}{8}$
$\frac{-Pb^2(L+2a)}{L^3}$	$\frac{-Pab^2}{L^2}$		$\frac{-Pa^2(L+2b)}{L^3}$	$\frac{Pa^2b}{L^2}$
$-P$	$-\alpha(1-\alpha)PL$		$-P$	$\alpha(1-\alpha)PL$
$\frac{-wL}{2}$	$\frac{-wL^2}{12}$		$\frac{-wL}{2}$	$\frac{wL^2}{12}$
$\frac{-7wL}{20}$	$\frac{-wL^2}{20}$		$\frac{-3wL}{20}$	$\frac{wL^2}{30}$

(Logan D L- A First Course in the Finite Element Method)