NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF INDUSTRIAL TECHNOLOGY

DEPARTMENT OF CIVIL AND WATER ENGINEERING

FINITE ELEMENT METHOD IN CIVIL ENGINEERING

TCW 5004

Examination Paper

December 2015

This examination paper consists of 7 pages

Time Allowed: 3 hours

Total Marks: 100

Special Requirements:

Examiner's Name: Miss Diana Makweche

INSTRUCTIONS

1. Answer any four (4) questions
2. Each question carries 25 marks
3. Use of calculators is permissible

MARK ALLOCATION

QUESTION	MARKS
1.	**25**
2.	**25**
3.	**25**
4.	**25**
5.	**25**
TOTAL	**100**

QUESTION 1

(a) Using the principle of Minimum Potential Energy;

(i) Formulate the global stiffness matrix for the system in Figure Q1.

(ii) Solve for the unknown displacements

(iii) Calculate the unknown forces and reactions. [10]

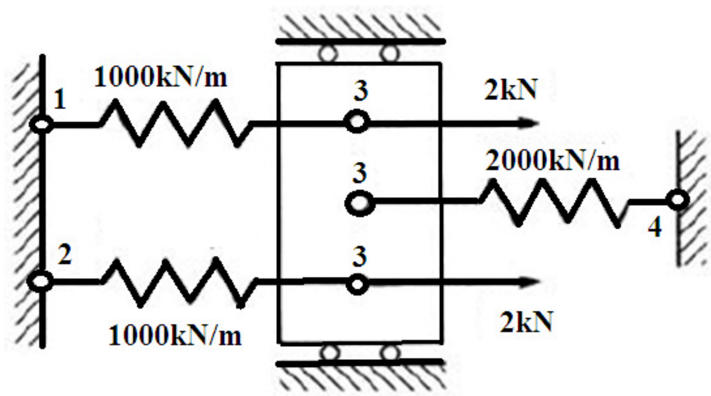

Figure Q1

(b) A bar of length l is subjected to a uniformly distributed load of intensity q. The potential energy of the system can be expressed as

$$\Pi\big(u(x)\big) = U - W = \frac{1}{2}\int_0^l EA\left(\frac{du}{dx}\right)^2 dx - \int_0^l qudx$$

Using the trial function $\mathrm{u}(x) = ax$,

(i) Determine the coefficient a

(ii) Find expressions for strain, ε, and stress, σ. [15]

QUESTION 2

(a) The bar element has two nodes and the linear displacement function is as shown in Figure Q2A.

(i) What are the equations for linear interpolation (shape) functions N_1 and N_2 ?

(ii) Sketch N_1 and N_2

(iii) What is the purpose of a shape function? [15]

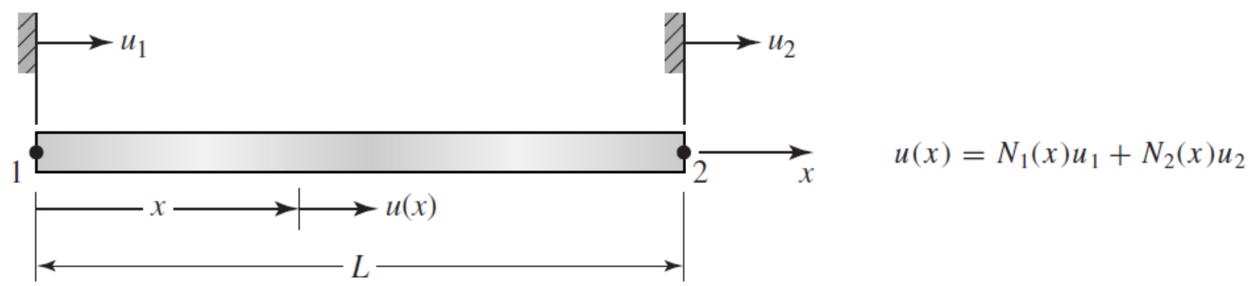

Figure Q2A

(b) (i) Explain how you would apply a fixed support to node 1 of the beam shown in Figure Q2B.

(ii) Reduce the uniformly distributed load into the equivalent nodal forces and moments. Give the answer in the form of the global force vector **F**. [10]

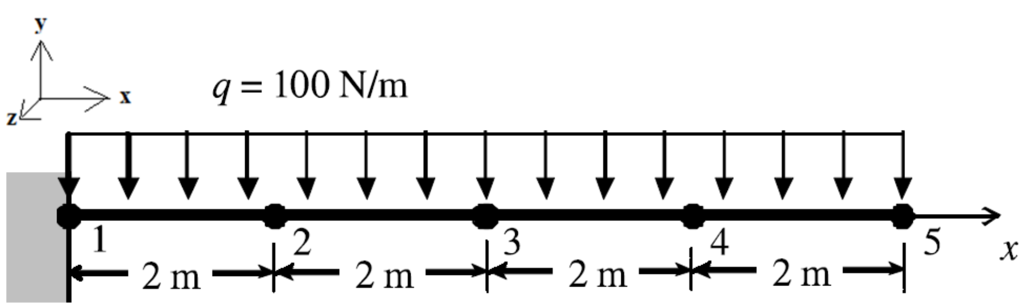

Figure Q2B

QUESTION 3

The tapered bar in Figure Q3 has a linearly varying cross-sectional area which varies from $3A_0$ at the left side to A_0 at the right side. The bar is divided into two elements. Show that the stiffness matrix for the system is $\begin{bmatrix} \frac{3k}{2} & \frac{-3k}{2} & 0 \\ \frac{-3k}{2} & 4k & \frac{-5k}{2} \\ 0 & \frac{-5k}{2} & \frac{5k}{2} \end{bmatrix}$ $where\ k = \frac{EA_0}{L}$.

Find a solution by integrating $\sigma_x = E\left(\frac{du}{dx}\right) = \frac{P}{A}(x)$subject to the boundary condition $u(0) = 0$

[25]

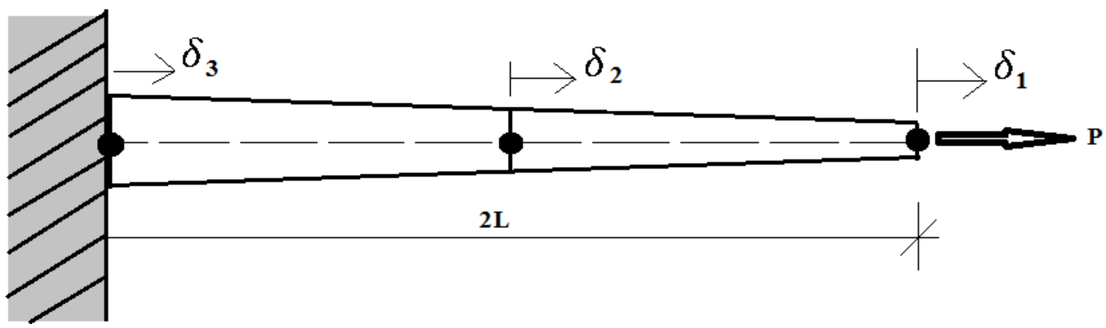

Figure Q3

QUESTION 4

(a) What is the difference between a beam and a bar (rod) element. [3]

(b) The truss in Figure Q4 is composed of two members each of cross-sectional area 968mm2. E =70GPa.

(i) Construct the individual elemental matrices

(ii) Assemble the global matrix [22]

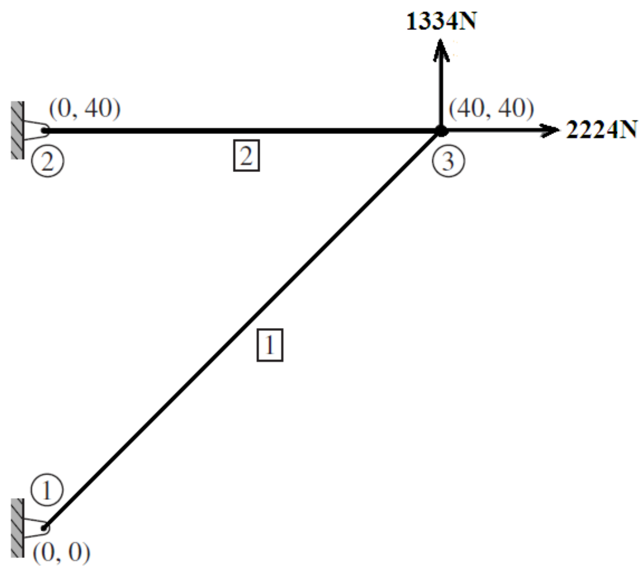

Figure Q4

QUESTION 5

The structure in Figure Q5 is subjected only to gravitational forces acting on the elements.

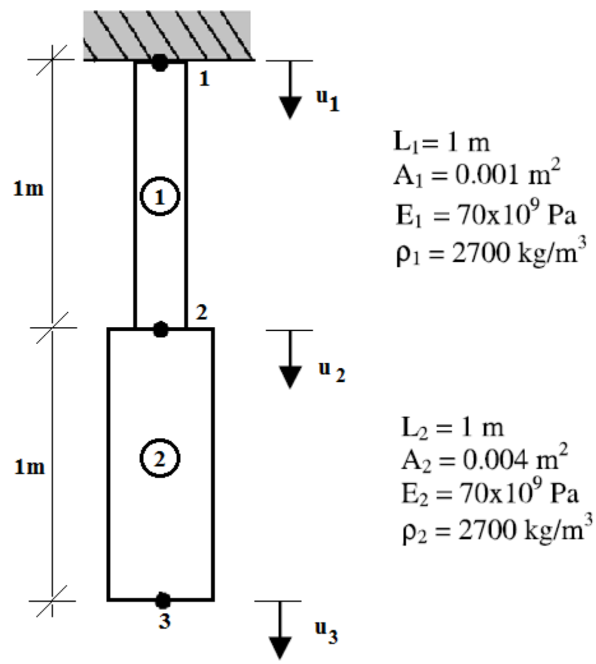

Figure Q5

(i) Derive the stiffness matrix for each element and assemble the global stiffness matrix

(ii) Construct the force vector considering the gravitational force acting on the elements

(iii) Apply the relevant boundary conditions and solve for the nodal displacements

(iv) Calculate the reaction force at the support, element strains and element stresses.

[25]

FORMULAE

$$\begin{Bmatrix} \hat{f}_{1x} \\ \hat{f}_{2x} \end{Bmatrix} = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} \hat{d}_{1x} \\ \hat{d}_{2x} \end{Bmatrix}$$

$$\underline{k} = \frac{AE}{L}\begin{bmatrix} C^2 & CS & -C^2 & -CS \\ & S^2 & -CS & -S^2 \\ & & C^2 & CS \\ \text{Symmetry} & & & S^2 \end{bmatrix}$$

$$\begin{Bmatrix} \hat{d}_x \\ \hat{d}_y \end{Bmatrix} = \begin{bmatrix} C & S \\ -S & C \end{bmatrix}\begin{Bmatrix} d_x \\ d_y \end{Bmatrix}$$

$$\begin{Bmatrix} \hat{f}_{1y} \\ \hat{m}_1 \\ \hat{f}_{2y} \\ \hat{m}_2 \end{Bmatrix} = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}\begin{Bmatrix} \hat{d}_{1y} \\ \hat{\phi}_1 \\ \hat{d}_{2y} \\ \hat{\phi}_2 \end{Bmatrix}$$