NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF INDUSTRIAL TECHNOLOGY

DEPARTMENT OF CIVIL AND WATER ENGINEERING

FINITE ELEMENT METHOD IN CIVIL ENGINEERING

TCW 5004

Supplementary Examination Paper

JULY 2016

This examination paper consists of 5 pages

Time Allowed: 3 hours

Total Marks: 100

Special Requirements:

Examiner's Name: Miss Diana Makweche

INSTRUCTIONS

1. Answer any four (4) questions
2. Each question carries 25 marks
3. Use of calculators is permissible

MARK ALLOCATION

QUESTION	MARKS
1.	**25**
2.	**25**
3.	**25**
4.	**25**
TOTAL	**100**

QUESTION 1

The elastic rod in Figure Q1 is to be modeled using 3 bar elements. It is attached to a fixed support at one end and the other is subjected to a tensile force P. It has length L and the cross-sectional area varies linearly from A_0 at the support to $\frac{A_0}{2}$ at the loaded end.

(i) With the aid of clearly labeled diagrams, explain how you would idealize the bar to account for the varying cross-section [10]

(ii) Write expressions for the constraint at the fixed support (clearly indicate axes on diagram). [5]

(iii) Determine the stiffness matrix for each element [10]

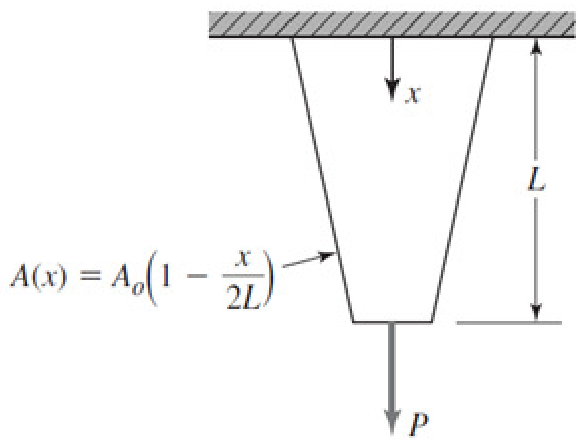


Figure Q1

QUESTION 2

Using the Principle of Minimum Potential Energy

(i) Derive the stiffness matrix for the system of linear springs in Figure Q2.

(ii) Solve for the displacements and reaction forces given that

$k_1 = 4N/mm,\ k_2 = 6N/mm,\ k_3 = 3N/mm,\ F_2 = -30N, F_3 = 0, F_4 = 50N$ [25]

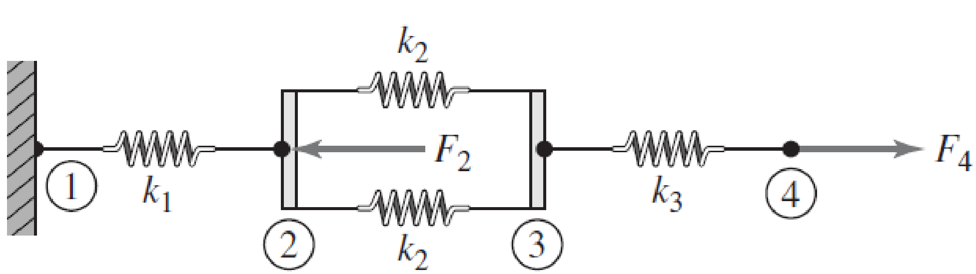


Figure Q2

QUESTION 3

(a) Consider the element in Figure Q4. The coordinates are in millimetres. $E = 210GPa$, $v = 0.25,\ t = 10mm$

(i) Evaluate the stiffness matrix for the element assuming plane stress conditions.

(ii) the nodal displacements are $u_1 = 2.0mm, v_1 = 1.0mm, u_2 = 0.5mm,$ $v_2 = 0.0mm, u_3 = 3.0mm, v_3 = 1.0mm.$

Determine the element stresses σ_x, σ_y and τ_{xy} [25]

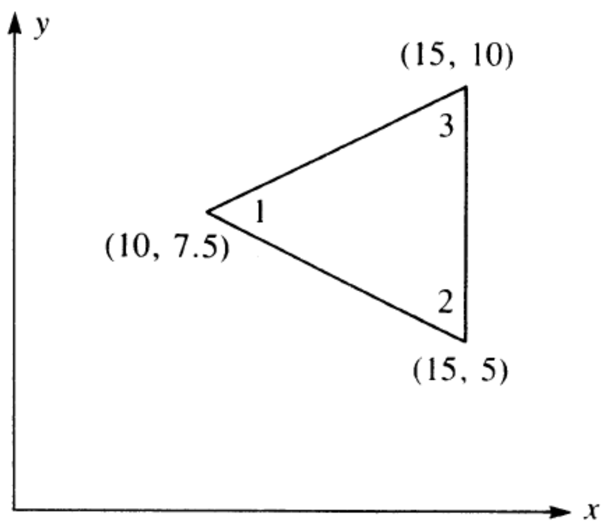


Figure Q3

QUESTION 4

The propped cantilever in Figure Q4 is subjected to an end load P. The beam is of length 2L and has constant EI. Determine the nodal displacements and reaction forces. [25]

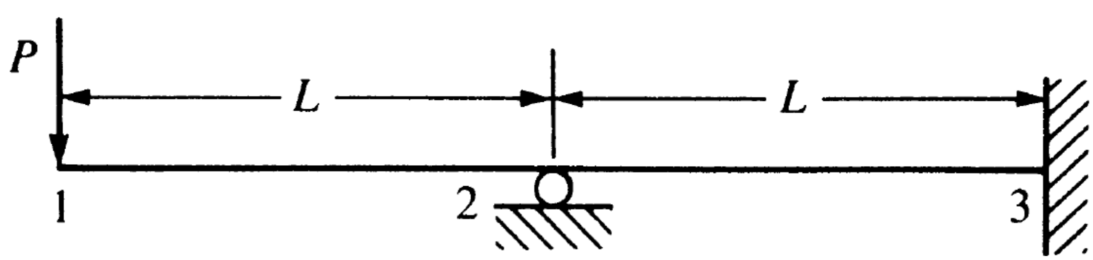


Figure Q4

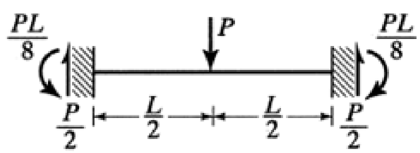
FORMULAE

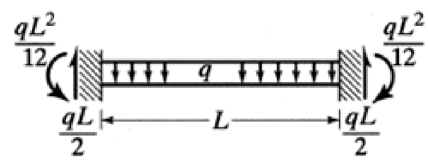
$$\begin{Bmatrix} \hat{f}_{1x} \\ \hat{f}_{2x} \end{Bmatrix} = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} \hat{d}_{1x} \\ \hat{d}_{2x} \end{Bmatrix}$$

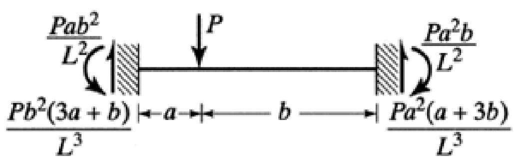
$$\underline{k} = \frac{AE}{L}\begin{bmatrix} C^2 & CS & -C^2 & -CS \\ & S^2 & -CS & -S^2 \\ & & C^2 & CS \\ \text{Symmetry} & & & S^2 \end{bmatrix}$$

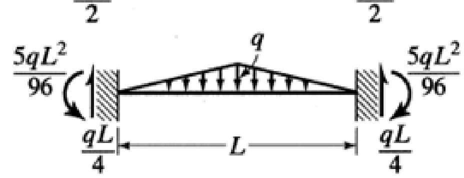
$$\begin{Bmatrix} \hat{d}_x \\ \hat{d}_y \end{Bmatrix} = \begin{bmatrix} C & S \\ -S & C \end{bmatrix}\begin{Bmatrix} d_x \\ d_y \end{Bmatrix}$$

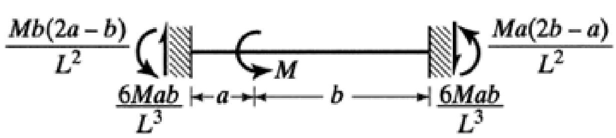
$$\begin{Bmatrix} \hat{f}_{1y} \\ \hat{m}_1 \\ \hat{f}_{2y} \\ \hat{m}_2 \end{Bmatrix} = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}\begin{Bmatrix} \hat{d}_{1y} \\ \hat{\phi}_1 \\ \hat{d}_{2y} \\ \hat{\phi}_2 \end{Bmatrix}$$

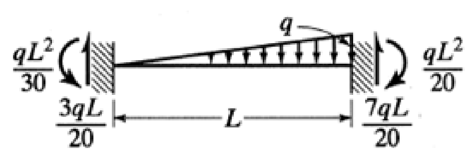












$$\mathbf{k} = \int_V \mathbf{B}^T\mathbf{D}\mathbf{B}\,dV = tA(\mathbf{B}^T\mathbf{D}\mathbf{B})$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{1-\nu^2}.\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}.\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{(1+\nu).(1-2.\nu)} \cdot \begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2.\nu}{2} \end{bmatrix} \cdot \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

$$A = \frac{1}{2} \det \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix}$$

$$\mathbf{B} = \frac{1}{2A} \begin{bmatrix} y_{23} & 0 & y_{31} & 0 & y_{12} & 0 \\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21} \\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12} \end{bmatrix}$$

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \mathbf{Bd} = \frac{1}{2A} \begin{bmatrix} y_{23} & 0 & y_{31} & 0 & y_{12} & 0 \\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21} \\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12} \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix}$$