NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF INDUSTRIAL TECHNOLOGY

DEPARTMENT OF CIVIL AND WATER ENGINEERING

FINITE ELEMENT METHOD IN CIVIL ENGINEERING

TCW 5004

Examination Paper

December 2016

This examination paper consists of 6 pages

Time Allowed: 3 hours

Total Marks: 100

Special Requirements: None

Examiner's Name: Miss Diana Makweche/ Mrs Faith Makwiranzou

INSTRUCTIONS

1. Answer any four (4) questions. Credit will not be given for additional questions attempted.
2. Each question carries 25 marks
3. Where relevant, use the solution method prescribed.

MARK ALLOCATION

QUESTION	MARKS
1.	**25**
2.	**25**
3.	**25**
4.	**25**
5.	**25**
TOTAL	**100**

QUESTION 1

(i) What is the Finite Element Method (FEM)? [3]

(ii) There are two main approaches to the FEM, the Mathematical and the Physical. Differentiate between the two. [4]

(iii) Name and give a brief description of the basic steps of the FEM? [12]

(iv) With reference to the finite element mesh, the accuracy of a solution obtained may be increased by employing different modeling strategies. Briefly outline three such strategies. [6]

QUESTION 2

(i) Elements may be linear (1D), two dimensional (2D), or three dimensional (3D). Sketch and name an example of each. [6]

(ii) What are *higher order elements*? [3]

A plate with a hole is subjected to tension as shown in Figure Q2A. In order to reduce the computational effort the problem is to be modeled exploiting symmetry.

(iii) What type of analysis is this? [2]

(iv) With the aid of a clearly labeled diagram, explain how you would mesh the **shaded** portion of the plate. [6]

(v) State the boundary conditions that would you apply to Line 1 and Line 2? [4]

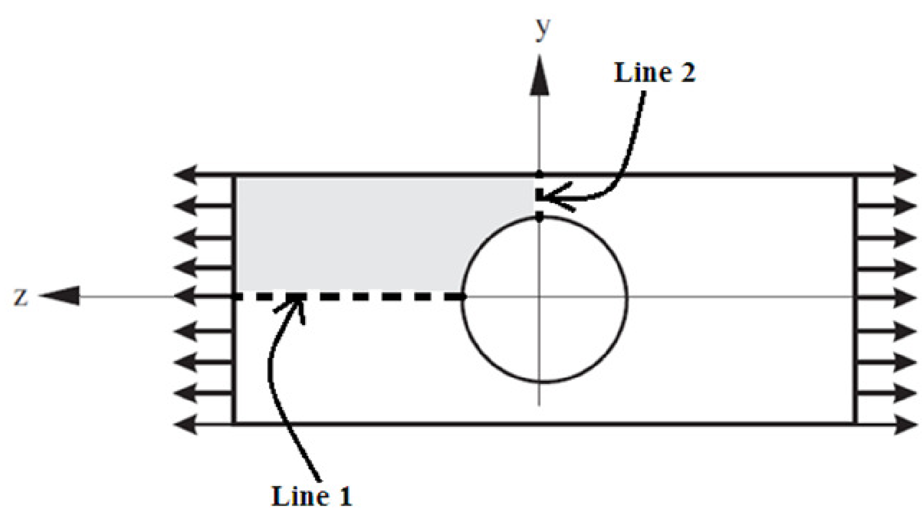


Figure Q2A

(vi) Figure Q2B shows two examples of a poor mesh. In each instance state why the mesh is not appropriate. [4]

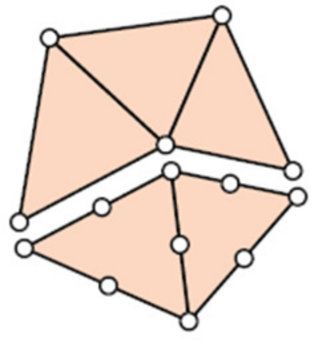


Figure Q2B

QUESTION 3

(i) State the Principle of Minimum Potential Energy. [3]

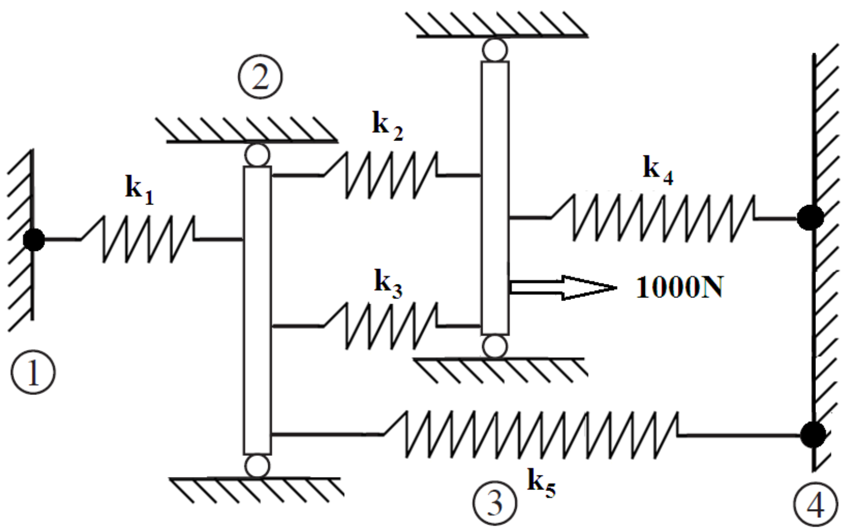


Figure Q3

Figure Q3 shows a spring-mass system.

$k_1 = 500N/\text{mm}$	$k_2 = 300N/\text{mm}$	$k_3 = 300N/\text{mm}$
$k_4 = 400N\ /\text{mm}$	$k_5 = 400N\ /\text{mm}$	$F_3 = 1000N$

(ii) Construct an expression for the total potential energy. [5]

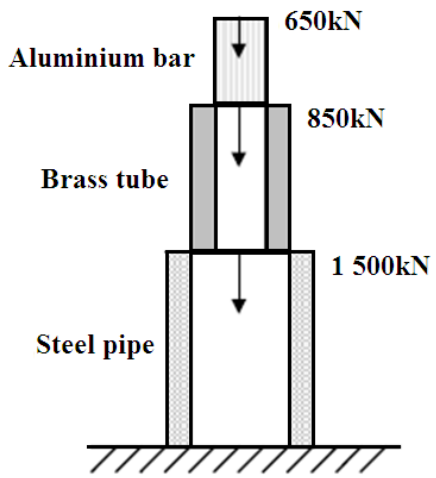
(iii) Minimise the expression with respect to each degree of freedom. [4]

(iv) Assemble the global system of equations. [5]

(v) Hence, find the displacements of nodes 2 and 3 and the reaction forces at nodes 1 and 4. [8]

QUESTION 4

Three metal tubes rest one on top of another as shown in Figure Q4.



Aluminium

$$E = 73GPa$$
$$diameter, D = 100x10^{-3}m$$
$$L = 1m$$

Brass

$$E = 100GPa$$
$$D_{outer} = 150x10^{-3}m$$
$$D_{inner} = 100x10^{-3}m$$
$$L = 1.25m$$

Steel

$$E = 210GPa$$
$$D_{outer} = 200x10^{-3}m$$
$$D_{inner} = 125x10^{-3}m$$
$$L = 0.75m$$

$$NB: A = \frac{\pi}{4}\left(D_{outer}^2 - D_{inner}^2\right)m^2$$

Figure Q4A

(i) Idealise the system as a spring-mass system. [3]

(ii) Calculate the stiffness of each element. [6]

(iii) Assemble the global stiffness matrix. [6]

(iv) Find the displacements at the nodes where the forces are applied. [6]

(v) Briefly describe how you would model this problem using a software package of your choice. Use sketches where necessary. (Include such details as analysis type, element type, cross-section, material model, load and fixity type.) [4]

QUESTION 5

(i) What is a *shape function*? [3]

(ii) Sketch the shape functions for a beam element. [4]

(iii) State the assumptions that are made in the derivation of the beam element. [4]

The simply supported beam of Figure Q5 has a Young's Modulus, E , and moment of inertia, I. It is subjected to a uniformly distributed load of intensity q.

(iv) Assuming two elements of equal length, calculate the equivalent nodal forces. [2]

(v) Derive the stiffness matrix for the system. [6]

(vi) Determine the mid-span deflection. Assume $v_1 = v_3 = \theta_2 = 0$ [4]

(vii) The theoretical maximum deflection at mid-span is $\frac{5qL^4}{384EI}$. Compare the calculated deflection to the theoretical value. [2]

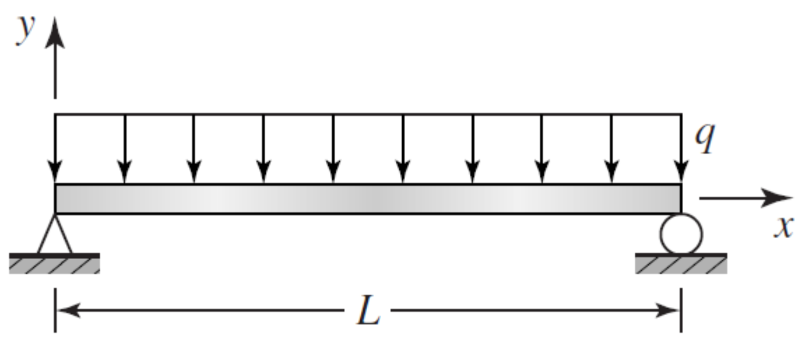
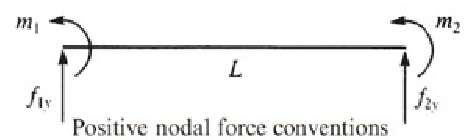


Figure Q5

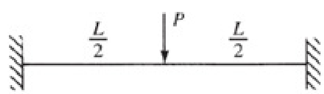
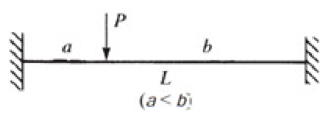
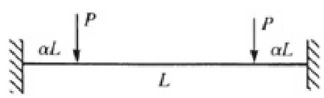
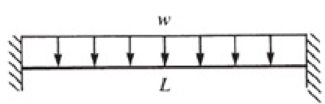
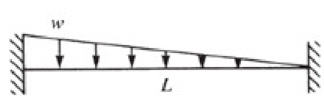
Formulae

$$\begin{Bmatrix} \hat{f}_{1y} \\ \hat{m}_1 \\ \hat{f}_{2y} \\ \hat{m}_2 \end{Bmatrix} = \frac{EI}{L^3} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \begin{Bmatrix} \hat{d}_{1y} \\ \hat{\phi}_1 \\ \hat{d}_{2y} \\ \hat{\phi}_2 \end{Bmatrix}$$



Positive nodal force conventions

Equivalent Nodal Forces for Different Load Types

f_{1y}	m_1	Loading case	f_{2y}	m_2
$\frac{-P}{2}$	$\frac{-PL}{8}$		$\frac{-P}{2}$	$\frac{PL}{8}$
$\frac{-Pb^2(L+2a)}{L^3}$	$\frac{-Pab^2}{L^2}$		$\frac{-Pa^2(L+2b)}{L^3}$	$\frac{Pa^2b}{L^2}$
$-P$	$-\alpha(1-\alpha)PL$		$-P$	$\alpha(1-\alpha)PL$
$\frac{-wL}{2}$	$\frac{-wL^2}{12}$		$\frac{-wL}{2}$	$\frac{wL^2}{12}$
$\frac{-7wL}{20}$	$\frac{-wL^2}{20}$		$\frac{-3wL}{20}$	$\frac{wL^2}{30}$