

# NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

## FACULTY OF INDUSTRIAL TECHNOLOGY BACHELOR OF ENGINEERING (HONS) DEGREE

Examination January 2013

### TEE3101 Digital Signal Processing

Duration of Examination 3 Hours

Instructions to Candidates:

1. Answer any **five** questions only.
2. Each question carries equal marks.
3. Show all your steps clearly in any calculation.
4. Start the answers for each question on a fresh page.

#### Question 1

Explain with the help of sketches and mathematical expression the complete characterisation of a discrete time signal in a linear time invariant system in terms of a unit impulse response .

( 20 marks)

#### Question 2

The transfer function of a discrete time system has poles at  $z=0.5$ ,  $z=0.1 +j0.2$ ,  $z=0.1-j0.2$  and zeros at  $z=-1$  and  $z=1$ .

- ( i ) Sketch the pole-zero diagram for the system.
- ( ii ) Derive the system transfer function from the pole –zero diagram.
- ( iii ) Develop the difference equation. (20 marks )

#### Question 3

- (a) Draw the block diagram that would represent a hardware architecture for a special purpose digital signal processor. Give an explanation of why the type of architecture shown by the diagram is chosen. (14 marks )
- (b) State at least four characteristics that would be included in the specification of a digital filter.

(4 marks)

- (c) Give the major difference between the finite impulse response digital filter and the infinite impulse response filter. (2 marks)

#### Question 4

- (a) The transfer function for a filter is given by

$H(z) = 1 - 1.3435z^{-1} + 0.9025z^{-2}$ . Draw the realisation block diagram for each of the following .

- (i) transversal structure,  
(ii) two lattice structure, and calculate the values of the coefficient for a lattice structure. (15 marks)
- (b) Give the five design steps for digital filters. (5 marks)

#### Question 5

- (a) Determine the discrete-time signal  $x[n]$  obtained from uniformly sampling at 400 Hz a continuous time signal  $x(t)$  given below;  
 $x(t) = 10\cos(120\pi t) + 6\sin(600\pi t) + 4\cos(680\pi t) + 8\cos(1000\pi t) + 12\sin(1320\pi t)$  (6 marks)
- (b) Using  $N=8$  explain the eight-point decimation-in-time FFT. Draw the butterfly diagram for the computation. (14 marks)

#### Question 6

- (a) Give three reasons that justify the use of oversampling in digital processing. (6 marks)
- (b) Discuss the use of uniform and non-uniform quantization and encoding. Give examples of the typical application of each technology. (14 marks)

**Question 7**

(a) Find the inverse Laplace transform of

(i) 
$$X(s) = \frac{2s^2 + 11s + 19}{(s+1)(s+2)(s+3)}$$

(ii) 
$$X(s) = \frac{2(3s^2 - 1)}{(s^2 + 1)^3} \quad (20 \text{ marks})$$

**Question 8**

(a) Find the z-transform of the following signals

(i)  $x[n] = na^{n-1}$

(ii)  $x[n] = 3\delta[n] + 5\delta[n-3] + 7\delta[n-8]$

(iii)  $x[n] = 3^n + (-1 + 0.8n)5^n$

(12 marks)

(b) Find the discrete signal corresponding to the z-transform

(i) 
$$X(z) = \frac{3}{1 - \frac{1}{2}z^{-1}} + \frac{2}{1 - \frac{1}{3}z^{-1}}$$

(ii) 
$$X(z) = \frac{Z^3}{(z - \frac{1}{2})(z + \frac{1}{3})^2}$$

(8 marks)

SOME COMMON z-TRANSFORM PAIRS

Transform pair	Signal	Transform	ROC
1.	$\delta[n]$	1	All $z$
2.	$u[n]$	$\frac{1}{1 - z^{-1}}$	$ z  > 1$
3.	$u[-n - 1]$	$\frac{1}{1 - z^{-1}}$	$ z  < 1$
4.	$\delta[n - m]$	$z^{-m}$	All $z$ except 0 (if $m > 0$ ) or $\infty$ (if $m < 0$ )
5.	$\alpha^n u[n]$	$\frac{1}{1 - \alpha z^{-1}}$	$ z  >  \alpha $
6.	$-\alpha^n u[-n - 1]$	$\frac{1}{1 - \alpha z^{-1}}$	$ z  <  \alpha $
7.	$n\alpha^n u[n]$	$\frac{\alpha z^{-1}}{(1 - \alpha z^{-1})^2}$	$ z  >  \alpha $
8.	$-n\alpha^n u[-n - 1]$	$\frac{\alpha z^{-1}}{(1 - \alpha z^{-1})^2}$	$ z  <  \alpha $
9.	$[\cos \Omega_0 n] u[n]$	$\frac{1 - [\cos \Omega_0] z^{-1}}{1 - [2 \cos \Omega_0] z^{-1} + z^{-2}}$	$ z  > 1$
10.	$[\sin \Omega_0 n] u[n]$	$\frac{[\sin \Omega_0] z^{-1}}{1 - [2 \cos \Omega_0] z^{-1} + z^{-2}}$	$ z  > 1$
11.	$[r^n \cos \Omega_0 n] u[n]$	$\frac{1 - [r \cos \Omega_0] z^{-1}}{1 - [2 r \cos \Omega_0] z^{-1} + r^2 z^{-2}}$	$ z  > r$
12.	$[r^n \sin \Omega_0 n] u[n]$	$\frac{[r \sin \Omega_0] z^{-1}}{1 - [2 r \cos \Omega_0] z^{-1} + r^2 z^{-2}}$	$ z  > r$

$f(t)$	TABLE OF LAPLACE TRANSFORM	$F(s)$
$f_1(t) + f_2(t)$	Linearity	$F_1(s) + F_2(s)$
$f(t)$	Definition	$\int_0^{\infty} f(t)e^{-st} dt$
$Kf(t)$	Linearity	$KF(s)$
$\frac{df(t)}{dt}$	Differentiation	$sF(s) - f(0)$
$\frac{d^n f(t)}{dt^n}$	Differentiation	$s^n F(s) - s^{n-1} f(0) - \dots - \frac{d^{n-1} f(0)}{dt^{n-1}} \dots$
$\int_0^t f(t) dt$	Integration	$\frac{1}{s} F(s)$
$tf(t)$	Complex differentiation	$-\frac{dF(s)}{ds}$
$e^{-at} f(t)$	Complex translation	$F(s+a)$
$f(t-a)u(t-a)$	Real translation	$e^{-sa} F(s)$
$f(t)$	Periodic function	$\frac{F_1(s)}{1 - e^{-sT}}$
$\int_0^t x(\tau)h(t-\tau)$	Convolution	$H(s)X(s)$
$\delta(t)$		1
$u(t)$		$\frac{1}{s}$
$e^{-at}u(t)$		$\frac{1}{s+a}$
$\sin \beta t u(t)$		$\frac{\beta}{s^2 + \beta^2}$
$\cos \beta t u(t)$		$\frac{s}{s^2 + \beta^2}$
$e^{-at} \sin \beta t u(t)$		$\frac{\beta}{(s+a)^2 + \beta^2}$
$e^{-at} \cos \beta t u(t)$		$\frac{s+a}{(s+a)^2 + \beta^2}$
$tu(t)$		$\frac{1}{s^2}$
$t^n u(t)$		$\frac{n!}{s^{n+1}}$
$te^{-at}u(t)$		$\frac{1}{(s+a)^2}$
$t^n e^{-at}u(t)$		$\frac{n!}{(s+a)^{n+1}}$