NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF INDUSTRIAL TECHNOLOGY
DEPARTMENT OF ELECTONIC ENGINEERING
BACHELOR OF ENGINEERING (HONS) DEGREE

Final Examination January 2013

TEE 5141 MODERN CONTROL ENGINEERING

Duration of Examination – 3 Hours

INSTRUCTIONS TO CANDIDATES

1. Answer any FOUR questions only.
2. Each question carries 25 marks.
3. Show your steps clearly in any calculation.
4. Start the answers for each question on a fresh page.

Question 1

a) Obtain the transfer function of the following system:

$$\bar{A} = \begin{bmatrix} -3 & 1 & 0 \\ -2 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}, \bar{B} = \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}, \bar{C} = [1 \quad 0 \quad 0], \bar{D} = [0]$$ [5 marks]

b) Using a block diagram, show how a full order observer can be combined with a controller to provide state feedback. [7 marks]

c) In the system shown in figure Q1 (c), D(z) is an integral controller of the form $D(z) = \frac{\alpha z}{z-1}$. Draw the root loci for T = 0.5 seconds and find the value of α for which the system becomes unstable. [13 marks]

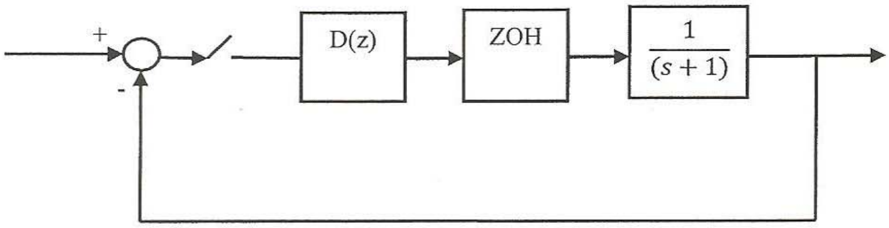

Figure Q1 (c)

Question 2

a) A system is given by the transfer function

$$\frac{3s^2-s+2}{s^3+2s^2+1}$$

Obtain the controllable canonical form state of the system and transform it using the matrix $P=\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}$. Comment on the result. [11 marks]

b) The transfer function of a system is given by $F(z)=\frac{Y(z)}{U(z)}=\frac{3}{1+0.5z^{-1}}$. Determine the output sequence for a unit step input. Draw the graph for y(k) for the case T = 1 second. What is the steady state value of the system? [7 marks]

c) The transfer function of a discrete system is $G(z)=\frac{z^2-2z+2}{z^3+2.1z^2+1.6z-0.4}$. Investigate the stability of the system. [7 marks]

Question 3

a) For the following transfer function, determine the Jordan Canonical form state matrices:

$$G(s)=\frac{2s^2+5s+1}{(s-1)(s^2-4s-4)}$$ [8 marks]

b) For the following system, obtain the transition matrix and hence determine the time response of the states:

$\bar{A}=\begin{bmatrix}1&1\\4&1\end{bmatrix}, \bar{B}=\begin{bmatrix}2\\1\end{bmatrix}, \bar{C}=[1\quad 1], \bar{D}=[0], \overline{x(0)}=\begin{bmatrix}3\\2\end{bmatrix}$ [8 marks]

c) The PID algorithm is given by

$$u(k)=u(k-1)+(k_p+k_I+k_D)e(k)-(k_P+2k_D)e(k-1)+k_De(k-2)$$

The algorithm is to be implemented in software by a computer. Using typical high level language programming statements, write a typical sequence of instructions to implement the algorithm. [9 marks]

Question 4

a) Describe use of eigenvectors in evaluation of the time response of a system. Give steps to be followed in the evaluation of the time response. [5 marks]

b) For the following dynamic system, determine a suitable full order asymptotic observer with poles at s = -1, -2.

$\bar{A}=\begin{bmatrix}2&0\\1&0\end{bmatrix}, \bar{B}=\begin{bmatrix}1\\0\end{bmatrix}, \bar{C}=[1\quad -1]$ [7 marks]

c) What do you understand by digital control? What are the advantages and disadvantages of digital control? [6 marks]

d) An under-damped, error-sampled, second-order system is required to have the following:

Damping ratio > 0.5
2% settling time < 10 seconds
A sampling frequency at least 8 times per damped oscillation period.

Sketch the region in the z-plane in which the system poles can be located. [7 marks]

Question 5

a) Give guidelines for choosing states in state space analysis. [2 marks]

b) Briefly outline situations where observers may be required. [2 marks]

c) Find the values of α for which the following system is controllable. Also test for its obsevability.

$$\bar{A} = \begin{bmatrix} \alpha & 0 \\ 0 & -3 \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \quad \bar{C} = \begin{bmatrix} 2 & 0 \end{bmatrix}$$

[8 marks]

d) In the system shown in figure Q5 (d), $D(z) = \frac{3-3z^{-1}+z}{1-z^{-1}}$, $G(s) = \frac{1}{s+1}$ and $H(s) = \frac{1}{s}$. Determine the transfer function for the system. [7 marks]

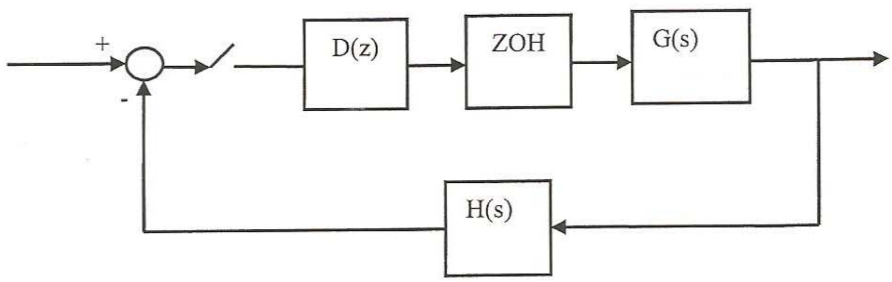

Figure Q5 (d)

e) Turn the analogue controller $G_c(s) = \frac{1+0.1s}{1-0.4s}$ into a digital algorithm using the bilinear transform method. [6 marks]

Question 6

a) What is the *'separation property'* in the context of combined state observer/state feedback design? [3 marks]

b) Give guidelines when selecting positions of poles for a system in feedback design. [2 marks]

c) Derive the state matrices for the circuit shown in figure Q6 (c), where $\bar{x} = [i_L \quad v_C]$, $\bar{u} = [i]$, and $\bar{y} = [V_o]$. x_1 is the current through the 0.2H inductor, x_2 is the voltage across the 0.1μF capacitor. [8 marks]

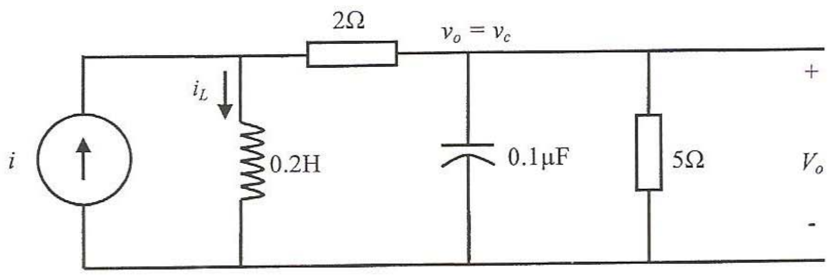

Figure Q6 (c)

d) What is aliasing, its effects and how can it be reduced? [4 marks]

e) For the sampled-data system shown in figure Q6 (e), the sampling period is 500 ms and the control algorithm is given by $u(k) = 2e(k) - 0.6u(k-1) - 0.4u(k-2)$. Find the position and velocity constants of the system. [8 marks]

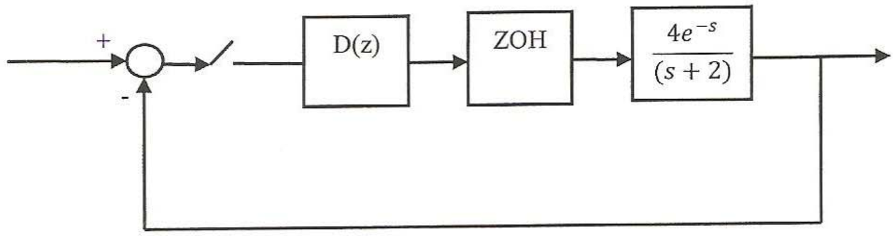

Figure Q6 (e)

END OF PAPER

National University of Science and Technology
Department of Electronic Engineering

Table of Laplace and z-Transform Pairs

f(t)	F(s)	f(k) = f(kT)	F(z)
1. impulse, $\delta(t)$	1	$\delta(k)$	1
2. unit step, u(t)	$\frac{1}{s}$	$u(k) = 1$	$\frac{z}{z-1} = \frac{1}{1-z^{-1}}$
3. shifted impulse, $\delta(t-nT)$	e^{-nsT}	$\delta(kT-nT)$	z^{-n}
4. unit ramp, t	$\frac{1}{s^2}$	kT	$\frac{Tz}{(z-1)^2}$
5. parabola, t^2	$\frac{2}{s^3}$	$(kT)^2$	$\frac{T^2 z(z+1)}{(z-1)^3}$
6. exponential, $e^{\pm at}$	$\frac{1}{s \mp a}$	$e^{\pm akT}$	$\frac{z}{z-e^{\pm aT}} = \frac{1}{1-e^{\pm aT}z^{-1}}$
7. $e^{-at} f(t)$	$F(s+a)$	$e^{-akT} f(kT)$	$F(z\, e^{aT})$
8.		c^k	$\frac{z}{z-c} = \frac{1}{1-cz^{-1}}$
9. $t\, f(t)$	$\frac{-dF(s)}{ds}$	$kT\, f(kT)$	$-Tz\frac{dF(z)}{dz}$
10. $\frac{1}{t} f(t)$	$\int_s^{\infty} F(s')\, ds'$	$\frac{1}{kT} f(kT)$	$-\frac{1}{T}\int \frac{F(z)}{z} dz$
11. $\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$	$\sin \omega kT$	$\frac{z \sin \omega T}{z^2 - 2z\cos \omega T + 1}$
12. $\cos \omega t$	$\frac{s}{s^2+\omega^2}$	$\cos \omega kT$	$\frac{z(z-\cos \omega T)}{z^2 - 2z\cos \omega T + 1}$
13. $e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$	$e^{-akT} \sin \omega kT$	$\frac{ze^{-akT} \sin \omega T}{[z-e^{(-a+j\omega\omega)}][z-e^{(-a-j\omega\omega)}]}$
14. $e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$	$e^{-akT} \cos \omega kT$	$\frac{z(z-e^{-akT} \cos \omega T)}{[z-e^{(-a+j\omega\omega)}][z-e^{(-a-j\omega\omega)}]}$
15. $\frac{e^{-at}-e^{-bt}}{b-a}$	$\frac{1}{(s+a)(s+b)}$		
16. $\frac{\omega}{\sqrt{(1-\zeta^2)}} e^{-\zeta\omega t} \sin\left[\omega\sqrt{(1-\zeta^2)}\,t\right]$	$\frac{\omega^2}{s^2+2\zeta\zeta\, s+\omega^2}$		
17. time advance, f(t+T)	$F(s)\, e^{+sT}$	$f(kT+T)$	$z[F(z)-f(0)]$
18. double time advance, f(t+2T)	$F(s)\, e^{+2sT}$	$f(kT+2T)$	$z^2[F(z)-f(0)]-zF(T)$
19. general time advance, f(t+nT)	$F(s)\, e^{+nsT}$	$f(kT+nT)$	$z^n F(z) - z^n f(0) - z^{-1} f(T) \cdots - zf(nT-T)$
20. general time retard, f(t-nT)	$F(s)\, e^{-nsT}$	$f(kT-nT)$	$z^{-n} F(z)$, one sided