NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF INDUSTRIAL AND MANUFACTURING ENGINEERING

TIE6130 OPERATIONS RESEARCH

NOV/DEC 2003
Time: 3hours

Answer all questions in section A and any THREE from section B.

Section A [40 marks] Answer all questions from this section

A1. COSD Company uses a single machine to process three jobs. Both the processing time and the due date (in days) for each job are given in the following table. The due dates are measured from the zero datum, the assumed start time of the first job.

Job	Processing Time (Days)	Due Date (Days)	Late Penalty ($/day)
1	5	25	19
2	20	22	12
3	15	35	34

The objective of the problem is to determine the minimum late-penalty sequence for the processing of the three jobs. Formulate as an integer linear programming model. **[7marks]**

A2. An investment broker has been given $1 000 000 to invest. He will choose the investment from a list of 25 stocks. The net return from 10 dollars in stock i is r_i. Given the risks and expected returns for the 25 stocks involved, the following operating policy will be adhered to:

a. No more than $125 000 will be invested in a single stock.
b. If any amount is invested in a stock, at least $25 000 will be invested in it.

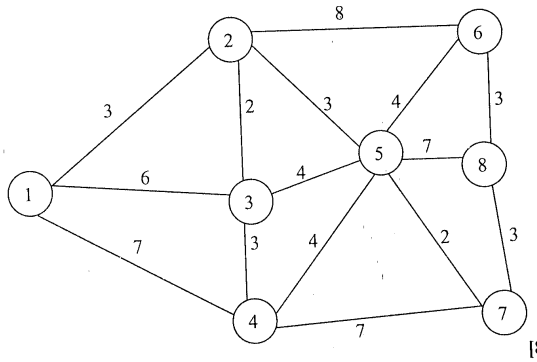
Formulate as a mixed integer linear programming problem that maximizes return. **[5marks]**

A3. A company employs service engineers based at various locations through out the country to service and repair their equipment installed in customers' premises. Four requests for service have been received and the company finds that four engineers are available. The distances in kilometers each of the engineers is from the various customers is given in the following table:

		Customers			
		W	X	Y	Z
Engineers	Takura	25	18	23	14
	Tatenda	38	15	53	23
	Tauya	15	17	41	30
	Tinotenda	26	28	36	29

The company wishes to assign engineers to customers to minimize the total distance to be travelled. Formulate as a linear programming model. **[8 marks]**

A4. A large manufacturing plant desires to connect computer terminals at seven locations across the plant site to the computer at its computing center. The network for the terminal locations are shown in the figure below. The values given along the arcs represent the distances in hundreds of metres between the terminal locations, which are denoted by the nodes 2 through 8. The plant manager wishes to determine how to connect all seven terminals with the computer center so that the minimum amount of cable is required. Find the shortest length of cable required.



[8marks]

A5. Amvest (Pvt) Ltd has $500 000 available for investment and seeks to determine how much of this amount is to be invested in each of four available investment alternatives: stock X, stock Y, bond X and bond Y. A maximum of $105 000 may be invested in bonds of type X and a maximum of $100 000 may be invested in bonds of type Y.
Annual net return is listed in the table below:

Annual Net Return On Investment	
Stock X	20%
Stock Y	10%
Bond X	9%
Bond Y	11%

Amvest is aware of the fact that there is a considerable amount of risk associated with investment in stock X. Therefore Amvest has determined that it will not invest an amount in stock X that exceeds one-fourth of its total investment. Also the total amount invested in stock Y must be at least three times the amount invested in stock X. Additionally, Amvest requires that its investment in bonds must be at least as great as one-half its investment in stocks. Management wants to maximize return and thus the total amount of money available after an investment period of one year. Formulate as a linear programming model. **[12marks]**

SECTION B: [60 marks]
Answer any THREE questions from this section

B6. A firm, for simplicity, is assumed to produce a single product. The firm has three plants (A, B, C), two warehouses (Y, Z) and four customers (1,2,3,4) geographically dispersed across the country. The firm ships its product from a plant to a warehouse and then to a customer. The information on supplies of this product at various plants, requirements of each customer and cost per unit of shipment from plants to warehouses and from warehouses to customer is provided below as follows:

Warehouse / Plants	Y	Z	Supply at Plant
A	18	23	25
B	19	21	29
C	25	16	16

Thus, total supply = 70.

Customer / Warehouse	1	2	3	4
Y	5	7	14	11
Z	12	15	10	8
Requirement	19	24	17	10

And total requirement = 70.
The firm wants to distribute its product at minimum cost.

(a) Define appropriate variables and formulate the above as a linear programming model.
(b) How is the LP model modified if supply at plant C is raised to 20 units?
(c) How would you modify the linear programming model if the amount shipped from plant B to warehouse Y had to be at least 7 and at most 12? **[20 marks]**

B7. A foundry produces three types of large metal castings. To produce casting A requires four hours, to produce casting B requires two hours and to produce casting C requires one hour. The foundry has regular time capacity of 40 hours per week. The current weekly demand for its castings are as follows:
Casting A----- 20 castings;

Casting B ----- 20 castings;
Casting C ----- 25 castings.
The manager of the foundry has established the following set of operating goals which are prioritized as follows:
P1: Satisfy the weekly demand for castings.
P2: Avoid weekly overtime operation of the plant beyond 10 hours.
P3: Satisfy the weekly needs of a new prospective customer for 15 "A" castings and 20 "C" castings.
P4: Avoid weekly underutilization of production capacity.
Formulate as a goal programming model. **[20marks]**

B8. Viddc Chemicals (Pvt) Ltd manufactures three types of chemicals. The company has contracted to supply the following amounts of the three chemicals:

Chemical	**Contracted Sales (kg)**
1	2000
2	3500
3	1800

Viddc Chemicals' production is limited by the availability of processing time in two chemical reactors. Each chemical must be processed first in reactor 1 and then in reactor 2. The following table provides the hours of processing time available next month for each reactor and the processing time required in each reactor per kg of each chemical:

Reactor Processing Times (hour/kilogram)				
	Chemical			Reactor
	1	2	3	Availabilities
Reactor 1	0.05	0.04	0.01	200 hours
Reactor 2	0.02	0.06	0.03	150 hours

Owing to the limited availability of reactor processing time Viddc Chemicals has insufficient capacity to meet its demand with in-house production. Consequently Viddc Chemicals must purchase some chemicals from vendors having excess capacity and resell them to its own customers. The following table provides each chemical's in-house production cost and outside purchase cost:

Chemical	In-House Production Cost ($/kg)	Outside Purchase Cost ($/kg)
1	2.50	2.80
2	1.75	2.50
3	2.90	3.25

Viddc Chemicals's objective is to fill its customers' orders with the cheapest combination of in-house production and outside purchases. In short the company must decide how much of each chemical to produce in-house and how much of each chemical to purchase outside.

(a) Let m_i = quantity of chemical i manufactured by the company. i = 1,2,3 and b_i = quantity purchased of chemical i , i = 1,2,3. Formulate as a linear programming problem.

(b) Use the computer output attached in appendix A to answer the following questions. In some parts, the computer output may provide insufficient information to allow you to answer in which case you should state, 'Insufficient Information'.

(i) Viddc Chemicals anticipates an increase in inhouse production cost per kilogram of chemical 1 from \$2.50 to \$2.54. If this occurs, what are the optimal values of the decision variables and what is the optimal objective function value?

(ii) Viddc Chemicals has contracted a new vendor to supply with chemical 2 at a cost of \$2.40 per kilogram. With this new purchase cost, what are the optimal values of the decision variables and what is the optimal objective function value?

(iii) Suppose that the contracted sales of chemical 1 is exactly 2300kg. What is the optimal objective function value?

(iv) Suppose that the contracted sales of chemical 1 is exactly 1900kg. What is the optimal objective function value?

(v) If the availability of reactor 2 increases from 150 hours to 170 hours, what is the optimal total cost?

If the availability of reactor 2 decreases from 150 hours to 140 hours, what is the optimal total cost? **[20marks]**

B9. Mr Sigauke, proprietor of the Sigauke Frame Company has a contract to be completed over four weeks which involves delivery of completed door and window frames to a new housing complex to satisfy the builder's schedule. The builder Mr Mlambo requires 400 sets of frames in week 1, 700 in week 2, 900 in week 3 and 1300 in week 4. (Each set contains 3 door and 8 window frames). The frames cannot be delivered early so any storage must be at Sigauke's premises which only has 1000 cubic metres of storage space, which are needed for work-in-process. The owner of the adjacent factory (which pickles onions) has some space which he is willing to let Mr Sigauke use, but he will have to hire an extra worker to transfer the frame sets into storage at a cost of \$60 per hour. He estimates that the worker could store the sets at a rate of 25 per hour. The Sigauke factory can produce 600 frames per week using existing staff without incurring any extra costs. Output can be increased by overtime which costs an extra C dollars per week for each extra 100 frame sets produced where the cost varies over the four weeks due to staff availability, C = \$450 in week 1. \$400 in week 2, \$500 in week 3 and \$700 in week 4. Find the optimal production schedule so that the builder's requirements are satisfied and Mr Sigauke's total costs are minimized. **[20marks]**

END OF EXAMINATION

APPENDIX A

Date: Thu Nov 06 10:13:07 2003

*** OPTIMUM SOLUTION SUMMARY ***

Title: Sable Chemicals
Final iteration No: 6
Objective value (min) =18225.0000

Variable	Value	Obj Coeff	Obj Val Contrib
x1	2000.0000	2.5000	5000.0000
x2	1833.3334	1.7500	3208.3335
x3	0.0000	2.9000	0.0000
x4	0.0000	2.8000	0.0000
x5	1666.6665	2.5000	4166.6660
x6	1800.0000	3.2500	5850.0000

Constraint	RHS	Slack(-)/Surplus(+)
1 (=)	2000.0000	0.0000
2 (=)	3500.0000	0.0000
3 (=)	1800.0000	0.0000
4 (<)	200.0000	26.6667-
5 (<)	150.0000	0.0000-

*** SENSITIVITY ANALYSIS ***

Objective coefficients -- Single Changes:

Variable	Current Coeff	Min Coeff	Max Coeff	Reduced Cost
x1	2.5000	-infinity	2.5500	0.0000
x2	1.7500	1.6000	1.8000	0.0000
x3	2.9000	2.8750	infinity	-0.0250
x4	2.8000	2.7500	infinity	-0.0500
x5	2.5000	2.4500	2.6500	0.0000
x6	3.2500	-infinity	3.2750	0.0000

Right-hand Side -- Single Changes:

Constraint	Current RHS	Min RHS	Max RHS	Dual Price
1 (=)	2000.0000	0.0000	2727.2726	2.7500
2 (=)	3500.0000	1833.3335	infinity	2.5000
3 (=)	1800.0000	0.0000	infinity	3.2500
4 (<)	200.0000	173.3333	infinity	0.0000
5 (<)	150.0000	40.0000	190.0000	-12.5000

Objective Coefficients -- Simultaneous Changes d:

Nonbasic Var	Optimality Condition
x3	-0.0250 + -0.5000 d5 + 1.0000 d6 + 0.5000 d2 - d3 <= 0
x4	-0.0500 + 1.0000 d1 + 0.3333 d5 + -0.3333 d2 - d4 <= 0
sx11	-12.5000 + -16.6667 d5 + 16.6667 d2 <= 0

Right-hand Side Ranging -- Simultaneous Changes D:

Basic Var	Value/Feasibilty Condition
x1	2000.0000 + 1.0000 D1 >= 0
x5	1666.6665 + 0.3333 D1 + 1.0000 D2 + -16.6667 D5 >= 0
x6	1800.0000 + 1.0000 D3 >= 0
x10	26.6667 + -0.0367 D1 + 1.0000 D4 + -0.6667 D5 >= 0
x2	1833.3334 + -0.3333 D1 + 16.6667 D5 >= 0

End of Solution Summary