NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

AQS2109

FACULTY OF THE BUILT ENVIRONMENT

DEPARTMENT OF QUANTITY SURVEYING

AQS2109:STATISTICS I

APRIL 2009 EXAMINATIONS

Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY THREE** questions from Sections B. Formulae for some well known probability distributions are given in the APPENDIX at the end of the paper.

SECTION A: Answer ALL questions in this section [40].

A1. Define the following terms and give one practical example for each.

(a) Mutually exclusive events. [2]

(b) The p-value of a test. [2]

(c) Independent events. [2]

A2. The number of typing errors made by a particular typist has a Poison distribution with an average of 4 errors per page. If more than 4 errors show on a given page, the typist must re-type the whole page. What is the probability that a certain page does not have to be re-typed? [5]

A3. The mean time taken by a sample of 64 Pretoria-Jo'burg car commuters to commute to work was found to be 45 minutes with a variance of 256 minutes. Using a 95% confidence interval, estimate the average time, μ, taken by all Pretoria-Jo'burg car commuters. [5]

A4. Events A and B are such that $P(A) = \dfrac{5}{12}$ $P(A/\bar{B}) = \dfrac{7}{12}$ and $P(A \cap B) = \dfrac{1}{8}$.

(a) Find $P(B)$. [4]

(b) Find $P(B/A)$. [2]

(c) Are events A and B independent? [2]

(d) Are events A and B mutually exclusive? [1]

A5. An insurance company sells insurance policies to 5 construction companies to cover on site accident injuries to workers. The probability that no accident leading to worker injury will occur on site in the year of insurance cover is $\frac{2}{3}$. Find the probability that in the year covered

(a) All companies will not make any claims. [3]

(b) At least 3 companies will not make any claims. [3]

(c) At least 2 companies will make claims. [4]

A6. Show that

$$p(x) = \frac{1}{1+\lambda}\left(\frac{\lambda}{1+\lambda}\right)^{x}, \qquad x = 0, 1, 2, 3, ...; \ \lambda > 0,$$

qualifies as a discrete probability function. [5]

SECTION B: Answer THREE questions in this section [60].

B7. (a) For a shipment cable, suppose that the specifications call for a mean breaking strength of $2000kg$. A sampling of the breaking strength of a number of segments of the cable has a mean breaking strength of $1955kg$ with a standard error of the mean of $25kg$.

(i) Using the 5% level of significance test the significance of the difference found [6]

(ii) Using the sampling results find a 90% confidence interval estimate of the population mean,μ. [4]

(b) The branch manager of Maposa Bank wishes to develop a simulation model in order to help schedule jobs through the bank. He has evaluated the completion times for all the different types of jobs. For one particular job, the times to completion can be represented by the following probability density function:

$$f(x) = \begin{cases} \frac{x}{8} - \frac{1}{4} & \text{for } 2 \leq x \leq 4 \\ \frac{10}{24} - \frac{x}{24} & \text{for } 4 \leq x \leq 10 \\ 0 & \text{elsewhere} \end{cases}$$

Find,

(i) The cumulative distribution function of X. [7]

(ii) $P(2.5 < X < 3.5)$. [3]

B8. (a) A dashboard warning light is supposed to flash red if a car's oil pressure is too low. On a certain model, the probability that the light will flash red when the oil pressure is low is 0.95. 2% of the time the light flashes when oil pressure is not low. If there is a 10% chance that the oil pressure is low, what is the probability that a driver needs to be concerned if the oil light goes on? [8]

(b) Consider the following table:

FIRM	**DEFECTIVE TUBES PER BOX OF 100 UNITS**				
	0	1	2	3 or more	Total
Supplier A	500	200	200	100	1000
Supplier B	320	160	80	40	600
Supplier C	600	100	50	50	800
	1420	**460**	**330**	**190**	**2400**

(i) If one box had been selected at random from this universe, what is the probability that the box would have come from supplier B?. [1]

(ii) If a box was selected at random, what is the probability that it would have no defectives and would have come from supplier A? [2]

(iii) Given that a box selected at random came from supplier B, what is the probability that it contained 1 or 2 defective tubes? [3]

(iv) If a box came from supplier A, what is the probability that the box would have 2 or less defective tubes? [3]

(v) It is known that a box selected at random has 2 defective tubes, what is the probability that it came from supplier C? [3]

B9. (a) (i) Show that the mean, $E(X)$, for the Geometric distribution is $\dfrac{1-p}{p}$. [4]

(ii) In a certain community the probability that a couple will have a boy is $\dfrac{1}{3}$. A certain couple has decided that it will continue giving birth up to and until they have a boy. What is the probability that the couple's 8^{th} child will be a boy. [4]

(b) Over a long period of time a certain drug has been effective in 30% of the cases in which it has been prescribed. If a doctor is now administering this drug to 4 patients,

(i) What is the probability that it will be effective for at least 3 of the patients? [4]

(ii) Find the mean, $E(X)$, and variance, $\text{Var}(X)$, of this distribution. [2]

(c) Suppose the number of phone calls going through an exchange during a certain 5 minute interval follows a Poisson distribution with parameter $\lambda = 5$.

(i) What is the probability that exactly 3 phone calls will be made during a 1 minute period? [3]

(ii) What is the probability that no phone call will be made during a 1 minute period? [1]

(iii) What is the mean and variance of the distribution? [2]

B10. (a) Define Type I and Type II errors. [2]

(b) Distinguish between independent and dependent samples. [4]

(c) To determine the effectiveness of an industrial safety programme, the following data were collected over a period of a year on the average weekly loss of man hours due to accidents in 12 plants, before and after the programme was put in operation:

Before: 50 87 37 141 59 65 24 88 25 36 50 35

After: 41 75 35 129 60 53 26 85 29 31 48 37

Use $\alpha = 0.05$ to test the null hypothesis that the safety programme is not effective against a suitable one sided alternative. [Assume the number of accidents across the plants are normally distributed and that the variance is the same before and after]. [14]

APPENDIX 2: Some Discrete and Continuous Probabilty Distributions

Distribution	Probability function	Mean $E(X)$	Variance $E[(x-E(x))^2]$
Discrete r.v. *probability functions*			
1.Bernoulli	$p_x = p, \text{ for } \quad x = 1$ $p_x = 1-p, \text{ for } x = 0$	p	$p(1-p)$
2.Binomial	$p_x = \binom{n}{x} p^x (1-p)^{n-x}$ for $\quad x = 0, 1, \cdots, n$	np	$np(1-p)$
3.Negative Binomial	$p_x = \binom{x-1}{k-1} p^k (1-p)^{x-k}$ for $\quad x = k, k+1, k+2, \cdots, n$	np	$np(1-p)$
4.Geometric	$p_x = p(1-p)^{x-1}$ for $\quad x = 1, 2, \cdots$	$\frac{1-p}{p}$	$\frac{1-p}{p^2}$
5.Poisson	$p_x = \dfrac{e^{-\theta}\theta^x}{x!}$ for $x = 0, 1, 2, \cdots$	θ	θ
Continuous r.v *density functions*			
6.Exponential	$f(x) = \lambda e^{-\lambda x}$ for $x > 0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
7.Uniform	$f(x) = \dfrac{1}{b-a}$ for $a \leq x \leq b$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
8.Normal	$f(x) = \dfrac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ for $-\infty < x < \infty$	μ	σ^2

END OF QUESTION PAPER