NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

AQS2109

FACULTY OF ARCHITECTURE AND QUANTITY SURVEYING

DEPARTMENT OF QUANTITY SURVEYING

AQS2109:STATISTICS I

FEBRUARY 2010 EXAMINATION

Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY THREE** questions from Sections B. List of formulae for some known distribution functions are given in the APPENDIX at the end of the paper.

SECTION A

A1. (a) Two fair coins are tossed.

(i) Illustrate the possible outcomes on a possibility space diagram. [2]

(ii) Find the probability that two heads are obtained. [2]

(b) Events A and B are such that $P(A) = \frac{19}{30}$, $P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{4}{5}$. Find $P(A \cap B)$. [2]

(c) Given that a heart is picked at random from a pack of 52 playing cards, find the probability that it is a picture card. [3]

A2. Define the following terms and give an example of each.

(a) random experiment. [1]

(b) permutation. [1]

(c) mutually exclusive event. [1]

(d) significance level. [1]

A3. (a) A steering committee for the BSc Hons in Q.S. consists of 2 Chairperson of Departments, 1 Coordinator, 2 Committee members and 2 ex-officio members. Suppose that a sub-committee of 4 members consisting of 1 member from each group is to be chosen. How many different sub-committees are possible? [2]

(b) Players in a state lottery can bet on any sequence of four one-digit numbers, 0 to 9, and no digit can be repeated. What is the probability that the winning number will be 1234? [3]

(c) A graduate seminar includes five students. The instructor assigns two As and three Bs, but on the day that he must submit his grade list to the college, he forgets his grade book at home. How many ways can he report two As and three Bs? [3]

A4. (a) Find the binomial expansion of

(i) $(1+x)^2$. [3]

(ii) $(a+b)^3$. [3]

(b) Show that the number of all possible subsets of the set $\{1, 2, ..., n\}$ is 2^n. [3]

A5. The following table shows the frequency distribution of students in a statistics class, classified by race and sex.

	SEX		
RACE	Boys	Girls	**Total**
Black	10	15	25
White	5	20	25
Total	15	35	50

Suppose that a student is randomly selected from the class. Compute the probability that a student is a girl given that she is white. [3]

A6. Suppose that a fair coin is tossed twice and let X represent the number of heads.

(a) List the elements of the sample space. [2]

(b) Hence determine the range space of X. [2]

(c) Determine the probability distribution of X. Express your answer in tabular form. [3]

SECTION B

B7. (a) Prove the following De Morgan's laws.

(i) $(A \cup B)' = A' \cap B'$. [5]

(ii) $(A \cap B)' = A' \cup B'$. [5]

(b) **(i)** Box I contains 3 red and 2 blue marbles, while Box II contains 2 red and 8 blue marbles. A fair coin is tossed. If the coin turns up heads a marble is chosen from Box I; if it turns up tails a marble is chosen from Box II. Find the probability that a red marble is chosen. [5]

(ii) Suppose the one who tosses the coin does not reveal whether it has turned up heads or tails (so that the box from which a marble was chosen is not revealed), but does reveal that a red marble was chosen. What is the probability that Box I was chosen (i.e. the coin turned up heads)? [5]

B8. (a) A random variable X is uniformly distributed in $a \leq x \leq b$ with density function:

$$f(x) = \begin{cases} \frac{1}{(b-a)} & ,a \leq x \leq b \\ 0 & \text{elsewhere} \end{cases}$$

Show that

(i) $\mu = \frac{1}{2}(a+b)$. [3]

(ii) $\sigma^2 = \frac{1}{12}(b-a)^2$. [4]

(b) Let X be a continuous random variable with p.d.f.

$$f_x(x) = \begin{cases} x+1 & ,-1 \leq x < 0 \\ 1-x & ,0 \leq x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

(i) Find the c.d.f. of X. [5]

(ii) Sketch the graph of $F_x(x)$. [3]

(c) The discrete r.v. X has the probability distribution shown in the table. Verify that $Var(2X+3) = 4Var(X)$. [5]

x	1	2	3	4
$P(X = x)$	$\frac{3}{8}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{4}$

B9. (a) The probability that a particular production line will produce a nondefective electronic component is 0.6. If one randomly selects 10 components from this production line, what is the probability of selecting the following?

(i) Four nondefective components. [2]

(ii) At most four nondefective components. [3]

(iii) At least eight nondefective components. [3]

(iv) Find the mean and variance of the r.v. [2,3]

(b) The lifetime of a particular brand of car battery distributed by a national chain of auto parts stores is approximately normally distributed, with a mean of 42 months and a standard deviation of 4 months. What percentage of the batteries will have lifetimes between 48 months and 50 months. [3]

(c) A machine fills packets with spice which are supposed to have a mean weight of 40 grams. A random sample of 36 packets is taken and the mean weight is found to be 42.2 grams with a standard deviation of 6 grams. Test at the 5% level of significance whether the mean weight is 40 grams. [4]

B10. An experiment was planned to compare the mean time (in days) required to recover from a common cold for persons given a daily dose of 4 grams of vitamin C versus those who were not given a vitamin supplement. A sample of 35 adults was randomly selected for each treatment category and the mean recovery times and standard deviations for the two groups were as follows:

	Treatment	
	No vitamin supplement	4g Vitamin C
Sample size	35	35
Sample mean	6.9	5.8
Sample standard deviation	2.9	1.2
Population mean	μ_1	μ_2

Suppose the research objective is to show that the use of vitamin C reduces the mean time required to recover from a common cold and its complications.

(a) Perform an appropriate test and draw your conclusions at the 5% level of significance. [12]

(b) Find the 95% confidence interval for the difference in the means of the two populations. [8]

APPENDIX 2: Some Discrete and Continuous Probabilty Distributions

Distribution	Probability function	Mean $E(X)$	Variance $E[(x - E(x))^2]$
Discrete r.v. *probability functions*			
1.Bernoulli	$p_x = p,$ for $x = 1$ $p_x = 1 - p,$ for $x = 0$	p	$p(1-p)$
2.Binomial	$p_x = \binom{n}{x} p^x (1-p)^{n-x}$ for $x = 0, 1, \cdots, n$	np	$np(1-p)$
3.Negative Binomial	$p_x = \binom{x-1}{k-1} p^x (1-p)^{x-k}$ for $x = k, k+1, k+2, \cdots, n$	np	$np(1-p)$
4.Geometric	$p_x = p(1-p)^{x-1}$ for $x = 1, 2, \cdots$	$\frac{1-p}{p}$	$\frac{1-p}{p^2}$
5.Poisson	$p_x = \dfrac{e^{-\theta}\theta^x}{x!}$ for $x = 0, 1, 2, \cdots$	θ	θ
Continuous r.v *density functions*			
6.Exponential	$f(x) = \lambda e^{-\lambda x}$ for $x > 0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
7.Uniform	$f(x) = \dfrac{1}{b-a}$ for $a \leq x \leq b$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
8.Normal	$f(x) = \dfrac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ for $-\infty < x < \infty$	μ	σ^2

END OF QUESTION PAPER