NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

AQS2109

FACULTY OF ARCHITECTURE AND QUANTITY SURVEYING

DEPARTMENT OF QUANTITY SURVEYING

AQS2109:STATISTICS I

AUGUST 2011 SUPPLEMENTARY EXAMINATION

Time : 3 hours

Candidates should attempt **ALL** questions from Section A (**40 marks**) and **ANY THREE** questions from Section B (**20 marks each**). List of formulae for some known distribution functions are given in the APPENDIX at the end of the paper.

SECTION A

A1. (a) A laboratory test is 95% effective in detecting a certain disease when it is infact present. However the test yields a 'false positive' result for 1% of the health persons tested. Suppose that 0.5% of the total population has the disease. Calculate the probability that a person has the disease given that the result is positive. [3]

(b) A coin is loaded so that the probability that it turns up heads is 2/3 and that it turns up tails is 1/3. Suppose the coin is flipped once and let

$$X = \begin{cases} 0 & \text{if tails} \\ 1 & \text{if heads} \end{cases}$$

(i) Determine the range space of X. [2]

(ii) Is X discrete or continuous? [2]

(iii) Find the probability that $X = 0$. [2]

A2. (a) A developer of a new subdivision offers prospective home buyers a choice of Tudor, rustic, colonial, and traditional exterior styling in ranch, two-storey, and split-level floor plans. In how many different ways can a buyer order one of these homes? [3]

(b) Players in a state lottery can bet on any sequence of four one-digit numbers, 0 to 9, and no digit can be repeated. What is the probability that the winning number will be 1234? [3]

(c) From 4 Quantity surveyors and 3 Architects, find the number of committees that can be formed consisting of 2 Quantity surveyors and 1 Architect? [4]

A3. Three items are selected at random from a manufacturing process. Each item is inspected and classified defective, D, or nondefective, N.

(a) Draw a tree diagram to represent the sample space. [4]

(b) If X is the random variable representing the number of defective items, write down the possible numerical values for the random variable X. [2]

(c) Assuming that the process produces 25 % defectives, write down the probability distribution of X. [5]

A4. Suppose that a fair coin is tossed twice and let X represent the number of heads.

(a) List the elements of the sample space. [2]

(b) Hence determine the range space of X. [2]

(c) Determine the probability distribution of X. Express your answer in tabular form. [3]

A5. The following table shows the frequency distribution of students in a statistics class, classified by race and sex.

	SEX		
RACE	Boys	Girls	**Total**
Black	10	15	25
White	5	20	25
Total	15	35	50

Suppose that a student is randomly selected from the class. Compute the probability that a student is a girl given that she is white. [3]

SECTION B

B6. (a) A random variable X is uniformly distributed in $a \leq x \leq b$ with density function:

$$f(x) = \begin{cases} \frac{1}{(b-a)} & ,a \leq x \leq b \\ 0 & \text{elsewhere} \end{cases}$$

Show that

(i) $\mu = \frac{1}{2}(a+b)$. [3]

(ii) $\sigma^2 = \frac{1}{12}(b-a)^2$. [4]

(b) Let X be a continuous random variable with p.d.f.

$$f_x(x) = \begin{cases} x+1 & ,-1 \leq x < 0 \\ 1-x & ,0 \leq x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

(i) Find the c.d.f. of X. [5]

(ii) Sketch the graph of $F_x(x)$. [3]

(c) The discrete r.v. X has the probability distribution shown in the table. Find $E(X)$and $E(X^2)$, and hence find $E[(2X+1)^2]$. [5]

x	-3	6	9
$P(X=x)$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

B7. (a) Suppose three different techniques for teaching introductory statistics are to be investigated. Technique 1 involves using computer-assisted instruction(CAI) in conjuction with lectures, technique 2 involves CAI only, and technique 3 involves lectures only. Random samples of 100 students are assigned to each of the three teaching techniques and their final grades are used to compare the methods. Do the data provide sufficient evidence to indicate that the distribution of final grades **depends** on the teaching technique employed? Test at the 0.10 level of significance. [8]

(b) Participants in a random sample of 10 professional football players are placed on a yoghurt-and banana diet for one month. The weights before and after one month on diet are as below:

Player	A	B	C	D	E	F	G	H	I	J
Weight Before	187	205	165	193	199	286	212	189	242	253
Weight After	175	193	167	190	197	240	210	189	221	255

Test at the 5% level of significance whether the diet has been effective. [12]

B8. (a) The probability that a particular production line will produce a nondefective electronic component is 0.6. If one randomly selects 10 components from this production line, what is the probability of selecting the following?

(i) Four nondefective components. [2]

(ii) At most four nondefective components. [3]

(iii) At least eight nondefective components. [3]

(iv) Find the mean and variance of the r.v. [2,3]

(b) An electrical firm manufactures light bulbs that have a length of life that is normally distributed with mean equal to 800 hours and a standard deviation of 40 hours. Find the probability that a bulb burns between 778 and 834 hours. [3]

(c) A machine fills packets with spice which are supposed to have a mean weight of 40 grams. A random sample of 36 packets is taken and the mean weight is found to be 42.2 grams with a standard deviation of 6 grams. Test at the 5% level of significance whether the mean weight is 40 grams. [4]

B9. The following data represent the salaries to the nearest $1,000 of a random sample of 20 Quantity Surveyors employed by the Zimbabwe Quantity Surveyors Association:

56	92	43	74	78
70	41	97	72	72
62	68	89	84	62
71	69	91	64	57

(i) Using the data,calculate the following:

(a) mean [2]

(b) median [1]

(c) mode [1]

(d) standard deviation [3]

(ii) Clearly explaining:

(a) Draw a Stem and leaf plot of the data, [4]

(b) Draw a histogram of the data, [5]

(c) Construct a box and whisker plot of the data. [4]

B10. An experiment was planned to compare the mean time (in days) required to recover from a common cold for persons given a daily dose of 4 grams of vitamin C versus those who were not given a vitamin supplement. A sample of 35 adults was randomly selected for each treatment category and the mean recovery times and standard deviations for the two groups were as follows:

	Treatment	
	No vitamin supplement	4g Vitamin C
Sample size	35	35
Sample mean	6.9	5.8
Sample standard deviation	2.9	1.2
Population mean	μ_1	μ_2

Suppose the research objective is to show that the use of vitamin C reduces the mean time required to recover from a common cold and its complications.

(a) Perform an appropriate test and draw your conclusions at the 5% level of significance. [12]

(b) Find the 95% confidence interval for the difference in the means of the two populations. [8]

APPENDIX 2: Some Discrete and Continuous Probabilty Distributions

Distribution	Probability function	Mean $E(X)$	Variance $E[(x - E(x))^2]$
Discrete r.v. *probability functions*			
1.Bernoulli	$p_x = p$, for $x = 1$ $p_x = 1 - p$, for $x = 0$	p	$p(1-p)$
2.Binomial	$p_x = \binom{n}{x} p^x (1-p)^{n-x}$ for $x = 0, 1, \cdots, n$	np	$np(1-p)$
3.Negative Binomial	$p_x = \binom{x-1}{k-1} p^x (1-p)^{x-k}$ for $x = k, k+1, k+2, \cdots, n$	np	$np(1-p)$
4.Geometric	$p_x = p(1-p)^{x-1}$ for $x = 1, 2, \cdots$	$\frac{1-p}{p}$	$\frac{1-p}{p^2}$
5.Poisson	$p_x = \dfrac{e^{-\theta}\theta^x}{x!}$ for $x = 0, 1, 2, \cdots$	θ	θ
Continuous r.v *density functions*			
6.Exponential	$f(x) = \lambda e^{-\lambda x}$ for $x > 0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
7.Uniform	$f(x) = \dfrac{1}{b-a}$ for $a \leq x \leq b$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
8.Normal	$f(x) = \dfrac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ for $-\infty < x < \infty$	μ	σ^2

END OF QUESTION PAPER