NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

CFE 5305

FACULTY OF COMMERCE

DEPARTMENT OF FINANCE

CFE 5305: FINANCIAL TIME SERIES ANALYSIS

NOVEMBER/DECEMBER 2015: EXAMINATION

Time : 3 hours

Candidates should attempt **FOUR** questions.

A1. (a) Give two reasons why the study of financial time series involves returns, instead of prices, of assets ?

(b) Suppose that $\{r_t\}$ is a financial time series given by

$$r_t = \mu + \sum_{i=0}^{\infty} \psi_i a_{t-i},$$

where μ is the mean, $\psi_0 = 1$ and $\{a_t\}$ is a white noise series. Show that the autocorrelation function ρ_l is given by

$$\rho_l = \frac{\sum_{i=1}^{\infty} \psi_i \psi_{i+l}}{1 + \sum_{i=1}^{\infty} \psi_i^2}, \quad l \geq 0.$$

(c) Consider an AR(2) model of the form

$$r_t = \phi_0 + \phi_1 r_{t-1} + \phi_2 r_{t-2} + a_t;$$

where $\phi_0,\ \phi_1,\ \phi_2$ are constants and a_t is a white noise series.
Deduce, showing all essential details, that the autocorrelation function ρ_l is given by

$$\begin{aligned} \rho_0 &= 1, \\ \rho_1 &= \frac{\phi_1}{1 - \phi_2}, \\ \phi_l &= \phi \rho_{l-1} + \phi_2 \phi_{l-2}, \quad l \geq 2. \end{aligned}$$

(d) Suppose that the simple return of a monthly bond index follows the $MA(1)$ model

$$R_t = a_t + 0.2 a_{t-1}, \quad \sigma_a = 0.025.$$

Assume that $a_{100} = 0.01$.

(i) Compute 1-step and 2-step ahead forecasts of the return at the forecast origin $t = 100$.

(ii) Estimate the standard deviation of the estimated forecast errors.

(iii) Compute the lag-1 and lag-2 autocorrelations of the return series.

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[2,5,6,4,4,4]

A2. (a) State and explain the four steps for building a volatility model for an asset return series.

(b) Derive multi step ahead forecasts for a GARCH(1,2) model at the forecast origin h.

(c) Consider the following ARCH(1) model

$$a_t = \sigma_t \epsilon_t, \quad \sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2,$$

where $\alpha_0 > 0$ and $\alpha_1 \geq 0$.

(i) Show that the fourth moment, m_4, of a_t is given by

$$m_4 = \frac{3\alpha_0^2(1+\alpha_1)}{(1-\alpha_1)(1-3\alpha_1^2)}.$$

(ii) Discuss any two implications of the result in Question 2 (c) (i).

[5,8,8,4]

A3. (a) A stock price is currently \$60 per share and follows the geometric Brownian motion

$$dP_t = \mu P_t dt + \sigma P_t dt.$$

Given that the average return and the sample standard deviation for the year 2010 are 0.0031 and 0.02215, respectively, estimate the parameters μ and σ assuming that there were 252 trading days in 2010.

(b) Assume that in Question A3(a) the expected return μ from the stock is 20% per annum and its volatility is 40% per annum.

(i) What is the probability distribution for the stock price in 2 years?

(ii) Obtain the mean and standard deviation of the distribution and construct a 95% confidence interval for the stock price.

[8,8,9]

A4. Considering the forward price F of a nondividend-paying stock, we have

$$F_{t,T} = P_t \exp\{r(T-t)\},$$

where r is the risk-free interest rate, which is constant, and P_t is the current stock price. Suppose P_t follows the geometric Brownian motion

$$dP_t = \mu P_t dt + \sigma P_t dB_t.$$

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(a) Determine a stochastic diffusion equation for $F_{t,T}$.

(b) Find an explicit expression for $F_{t,T}$.

(c) Find $E[P_t]$ and $Var[P_t]$.

[6,7,12]

A5. Let a_t be white noise with variance σ_a^2 and let $\mid \phi \mid < 1$ be a constant. Consider the process

$$\begin{aligned} r_1 &= a_1, \\ r_t &= \phi r_{t-1} + a_t; \quad t = 2; 3; ... \end{aligned}$$

(a) Find the mean and the variance of $\{r_t\}$. Is $\{t_t\}$ stationary ?

(b) Show that

$$corr(r_t, r_{t-h}) = \phi^h \left[\frac{var(r_{t-h}}{var(r_t)}\right]^{\frac{1}{2}}$$

for $0 \leq h < t$.

(c) Deduce that for large t,

$$var(r_t) \approx \frac{\sigma_a^2}{1-\phi^2}$$

and

$$corr(r_t, r_{t-h}) \approx \phi^h, \quad h \geq 0.$$

(d) Comment on how you could use these results to simulate n observations of a stationary Gaussian $AR(1)$ process from simulated iid $N(0,1)$ values.

(e) Now suppose that

$$r_1 = \frac{a_1}{\sqrt{1-\phi^2}}.$$

Investigate whether this process is stationary or not ?

[4,6,6,4,5]

END OF QUESTION PAPER