



**National University of
Science and Technology**
Think in Other Terms



FACULTY OF COMMERCE
DEPARTMENT OF FINANCE
BACHELOR OF COMMERCE HONOURS DEGREE IN
FINANCE
PART I 1ST SEMESTER FINAL EXAMINATION - JANUARY 2011
FINANCIAL MATHEMATICS 1 [CFI 1101]
TIME ALLOWED: 3HOURS

INSTRUCTIONS TO CANDIDATES

1. Attempt QUESTION ONE and ANY FOUR others
2. Start the answer to each full question on a fresh page of the answer sheet.
3. Questions may be written in any order, but must be legibly numbered.
4. Write legibly, showing all workings.

INFORMATION FOR CANDIDATES

The businesses in this question paper are intended to be fictitious

The paper contains SIX (6) QUESTIONS.

All questions carry equal marks [20 marks]

Question One (Compulsory)

- a) Let X denote the present value of an annuity consisting of payments of \$2,000 payable at the end of each of the next 8 years, valued using an interest rate of 8% *pa* convertible quarterly and let Y denote the present value of an annuity consisting of payments of \$4,000 payable at the end of every fourth year for the next 16 years, valued using an interest rate of 8% *pa* convertible half yearly. Calculate the ratio X / Y . [6 marks]
- b) Find the present value of an annuity payable annually in advance for a term of 20 years such that the payment is \$500 in year 1, \$550 in year 2, \$600 in year 3 *etc.* Assume a rate of interest of 5% *pa* for the first twelve years and 7% *pa* thereafter. [8 marks]
- c) On 31 December 2000 a loan was issued to be repaid over ten years by a level annuity payable quarterly in arrears on the last days of March, June, September and December. The amount of the annuity was calculated on the basis of an interest rate of 16% *pa* convertible quarterly. Given that the total interest paid in 2004 was \$3,216.96, find the amount of the quarterly repayments and the amount of the loan outstanding on 1 January 2005. [6 marks]

Question Two

- (a) On 1 September 2001, a company placed part of its assets with two fund managers. Manager P was given \$80,000 and Manager Q was given \$140,000. Both managers received a net cash flow of \$15,000 on 1 September 2002, bringing their total fund values to \$103,000 and \$183,000, respectively. A further net cash flow of \$20,000 was received by each manager on 1 September 2003. This brought their total fund values to \$143,600 and \$239,600, respectively. On 31 August 2004, the value of Manager P 's fund was \$172,320 and the value of Manager Q 's fund was \$263,560.
- (i) Calculate the time weighted rate of return and the money weighted rate of return for each Fund manager for the period from 1 September 2001 to 31 August 2004. [4; 6 marks]
- (ii) By examining the growth factors between cash flows, describe the performance of each manager over the three-year period. Hence, explain why the money-weighted rate of return for Manager P is higher than that of Manager Q . [4 marks]
- (iii) Comment briefly on the relative performance of the two fund managers. [2 marks]
- (b) Identify and explain two key weaknesses of the IRR method that are addressed by the Modified IRR method. [4 marks]

Question Three

An investor deposits \$2,000, and then withdraws level annual payments starting one year after the deposit was made. Immediately after the 11th annual drawing, the investor has \$400 left in the account. Find the amount of each withdrawal, given that the annual rate of interest is 8%. [3 marks]

(b) The following information is provided about two projects A and B:

Year (t)	0	1	2	3	4
Project A \$'000	-2000	1000	800	600	200
Project B \$'000	-2000	200	600	800	1200

The cost of capital for both projects is 10%

- i. Calculate the NPV and IRR for projects A and B [4; 6 marks]
- ii. Calculate the cross-over rate for projects A and B [3 marks]
- iii. Sketch the NPV profiles of projects A and B. [2 marks]
- iv. Highlight and explain any conflict in ranking projects A and B using the NPV and IRR criteria if the cost of capital is 6%. [2 marks]

Question Four

a) Find the present value as at 1 June 2004 of 36 monthly payments each of \$300 commencing on 1 January 2005, assuming a rate of interest of 10% *pa* convertible half yearly.

[6 marks]

b) Define the following annuities:

- i. Immediate annuity
- ii. Annuity due
- iii. Ordinary annuity
- iv. Perpetuity annuity

[9 marks]

c) The following data relate to the assets of a small trust fund:

<i>Date</i>	<i>Market value</i>
1 January 2004	\$25,000
1 April 2004	\$29,000
1 July 2004	\$30,000
1 October 2004	\$32,000
1 January 2005	\$31,500

The only cash-flow during 2004 that was not generated from the assets of the fund was an injection of \$5,000 on 31 March. Calculate:

- (i) The money-weighted rate of return for the fund for 2004 [3 marks]
- (ii) The time-weighted rate of return for the fund for 2004 [2 marks]

Question Five

- a) A businessman wishes to borrow an amount of \$10,000 for a term of 5 years. The agreed rate of interest is 8% *pa* effective for the first 3 years, and 6% *pa* effective thereafter. Repayments on the loan are made annually in arrears.
- Calculate the amount of the level annual repayment. [3 marks]
 - Draw up the loan schedule for the full five-year period. [4 marks]
 - What percentage of the loan has been repaid by the end of the third year? [2 marks]
 - Without doing any further calculations, explain how this percentage figure would alter if the rate of interest had instead been 6% *pa* for the first three years and 8% *pa* thereafter. [2 marks]
- b) Find the rate of interest convertible monthly corresponding to:
- An effective rate of 14.2% *pa* [2 marks]
 - A nominal rate of 11% *pa* convertible three times a year [2 marks]
- c) Find the accumulated value as at 1 January 2005 of a series of payments of \$200 paid every two years from 1 January 1965 to 1 January 2003 inclusive, which accumulate at 12% *pa*. [5 marks]

Question Six

- a) Find the effective annual rate of interest corresponding to:
- A nominal rate of 11% *pa* convertible half yearly [2 marks]
 - A nominal rate of interest of 12% *pa* convertible monthly [2 marks]
- b) An investor purchases a 91 day Treasury bill 16 days after issue at a discount rate of 10% and sells it later at a discount rate of 9.5%. What is the equivalent yield at purchase and realized yield at sale? [4 marks]
- c) A loan of \$50,000 is being repaid over a period of 10 years by a series of level monthly installments. Interest is charged on the loan at the rate of 8% *pa* effective.
- Calculate the monthly repayment. [2 marks]
 - Calculate the amount of interest paid in the first year. [3 marks]
 - After the payment at the end of 7 years, the borrower takes a 2-month payment break, *i.e.* he does not pay the next 2 monthly installments. Calculate the extra amount he has to pay each month in order to clear his debt by the end of the 10th year. [4 marks]
- d) A woman who has won a prize is offered a lump sum of \$100,000 to invest now or \$55,000 to invest at the end of this year and another \$55,000 to invest at the end of the following year. If all investments are assumed to earn 7% *pa*, which should she choose if she intends to withdraw the money after:
- 4 years
 - 2 years [3 marks]