National University of Science and Technology

Think in Other Terms

FACULTY OF COMMERCE

DEPARTMENT OF FINANCE

BACHELOR OF COMMERCE HONOURS DEGREE IN FINANCE

PART I 1ST SEMESTER FINAL EXAMINATION– DECEMBER 2014

FINANCIAL MATHEMATICS I [CFI 1101]

TIME ALLOWED: 3 HOURS

INSTRUCTIONS TO CANDIDATES

1. Answer any **FOUR (4)** questions.
2. Show all workings.
3. Write neatly and legibly.

INFORMATION FOR CANDIDATES

1. This paper contains **FIVE (5)** Questions.
2. Each full question carries a total of **25 marks** and part marks are indicated in brackets at the end of each part question.
3. This paper contains **SIX (6)** printed pages.
4. A formula sheet is attached at the back of the question paper.
5. Candidates may write on the question paper but shall not write in the answer booklet during reading time.

QUESTION ONE

(a) On 1 January 2014, an investor purchases an annuity that promises to pay $800 at the end of every month starting on the 31st of January 2019. The purchase price of the annuity is $39 098.07, valued at 12% p.a. effective. How many payments should the investor expect to receive? **[6 marks]**

(b) As an analyst in the corporate lending department of a large bank in Zimbabwe, you have been asked to review the pricing of a one-year loan that was recently approved by your bank's Credit Committee. The loan was approved at an interest rate of 20% p.a. and establishment fees of 4% (collected in advance). However, the client has approached the bank with an offer from a competitor bank for the same amount but at an interest rate of 16% p.a. and establishment fees of 6%. The client is asking the bank to reduce its interest rate to 19%; otherwise she would borrow from the competitor bank instead.

Required:

(i) Calculate the effective cost of the loan to the client based on the pricing approved by the Credit Committee. **[3 marks]**

(ii) Calculate the effective cost of the loan to the client based on the pricing of the competitor bank. **[3 marks]**

(iii) Calculate the effective cost of the loan to the client based on the client's proposed pricing. **[3 marks]**

(iv) All else equal, should your bank's Credit Committee approve the client's request? (Support your answer.) **[3 marks]**

(c) An investor is considering two ways of investing $20,000 for a period of 10 years:

(i) Option A offers 6% per annum convertible every 3 months;

(ii) Option B offers 6.4% per annum convertible every 6 months.

Required: Which is the better option? **[4 marks]**

(d) An annuity certain with payments of $200 at the end of each quarter is to be replaced by an annuity with the same term and present value, but with payments at the beginning of each month instead. Calculate the revised payments at 12% p.a. effective. **[3 marks]**

Total [25 marks]

QUESTION TWO

(a) John, a first year student at NUST has just won a lottery for a net dividend of $50 000. He is evaluating two options for investing his unexpected wealth. Option 1 involves investing the entire $50 000 in a bank savings account earning 12% p.a. convertible semi-annually. Option 2 involves purchasing an annuity for $50 000 that promises to pay $550 at the end of every month for 20 years, starting immediately. All else equal, which of the two options should John take? (Support your answer with relevant calculations). **[6 marks]**

(b) You have invested $100 000 in a 10-year project that is expected to generate net cash flow of $8 000 at the end of the first year, increasing by a constant dollar amount, Q every year thereafter. If the required return on the project is 12% p.a. effective, how much must be the annual dollar increment in net cash flow so that the project breaks even (i.e. has a net present value of zero)? **[6 marks]**

(c) A company needs to replace a machine costing $50,000 in 6 years' time. To achieve this it will make six annual investments, starting at the end of this year, earning interest of 5.5% per annum. Find the value of the annual payment. **[3 marks]**

(d) A recently-married couple take out a 25-year mortgage loan for $30 000 at 12% p.a. effective. The mortgage loan is repayable in equal monthly installments of principal and interest.

(i) What is the gross monthly installment? **[3 marks]**

(ii) What percentage of the mortgage loan will be outstanding after 10 years? **[3 marks]**

(iii) What percentage of the mortgage loan will be repaid between year 10 and year 16 (i.e. from start of year 11 to end of year 15)? **[4 marks]**

Total [25 marks]

QUESTION THREE

(a) An investor buys a 90 day Treasury bill at a discount of 6% per annum and holds it for 30 days before selling it to another investor at a discount of 5.6% per annum. The first investor invests the proceeds from the sale of the Treasury bill in a 60 day negotiable certificate of deposit (NCD) earning a yield of 5.8% per annum.

Required:

(i) Calculate the annualized yield earned by the first investor on the Treasury bill.

[4 marks]

(ii) How does his annualized yield over the entire 90 days compare to the annualized yield he would have earned if he had held the Treasury bill to maturity?**[5 marks]**

(b) In a letter dated 23 June 2014, CABS Building Society Managing Director, Mr. Kevin Terry advised CABS clients that: "On our mortgage loans we recently extended the term to 20 years. What this means is that the monthly repayment for a residential loan will be approximately $137 per month for every $10,000 borrowed." Calculate the effective annual cost of the mortgage loans referred to above. **[5 marks]**

(c) CAPM Building Society is offering 15-year mortgage loans to its employees at 15% per annum convertible monthly. In addition, CAPM collects 4% of the loan amount upfront as 'establishment fees'. John applies for a loan of $50,000 to purchase a house in the medium density suburbs of Bulawayo. Repayments are made as monthly installments of principal and interest over 15 years.

(i) Calculate the level monthly installment on John's loan. **[4 marks]**

(ii) Calculate the total amount of interest paid by John for the first 10 years.

[4 marks]

(d) Calculate the fair price of a 20-year, 10% coupon bond with a $100 face value and a yield to maturity of 12% per annum. Assume semi-annual payment of coupons.**[3 marks]**

Total [25 marks]

QUESTION FOUR

(a) Given a compound interest rate of 10% per annum convertible quarterly, calculate:

(i) The equivalent force of interest, δ. **[2 marks]**

(ii) The effective annual rate of commercial discount, d **[2 marks]**

(iii) The equivalent annual rate of interest convertible monthly, $i^{(12)}$. **[2 marks]**

(iv) The equivalent annual rate of discount convertible monthly, $d^{(12)}$. **[3 marks]**

(b) A University student is considering taking a 1-year bank loan for $1,000 to cover tuition fees. He approaches two banks, ABC and XYZ, and gets the following information:

	Tenure of loan	Establishment Fees	Annual Interest rate
ABC	1 year	5%	25%
XYZ	1 year	6.5%	23%

ABC requires that the student pays interest only at the end of each of the next 12 months, starting immediately, and then repays the principal amount as a lump sum at the end of the year. On the other hand, XYZ requires the student to make level payments of interest and principal at the end of each of the next 12 months, starting immediately.

Required:

Which of the two banks is providing the best deal to the student? Fully justify your answer with relevant calculations. **[6 marks]**

(c) Calculate the fair price of a 15-year, 10% coupon bond with a $100 face value and a yield to maturity of 12% per annum. Assume semi-annual payment of coupons**.[3 marks]**

(d) The bond in (c) above is trading at $97 and the time remaining to maturity is now 6 years. What is the approximate yield to maturity of the bond? **[2 marks]**

(e) Calculate the price of a zero coupon bond that has the same yield and term to maturity as the bond in (c) above **[2 marks]**

(f) An investment is discounted for 60 days at a simple rate of discount of 5% per annum. Calculate the annual effective rate of interest. **[3 marks]**

Total [25 marks]

QUESTION FIVE

(a) Jane and Esther are first year students at NUST. They have just signed a lease agreement with Mr. Peterson for the lease of a one-bedroomed flat in town for a period of four years. The terms of the lease are that rentals will be $350 per month, payable in advance. Rentals will increase by 10% after every 12 months for the entire term of the lease.

Required:

Calculate the present value of the expected lease payments at 12% p.a. effective.

[4 marks]

(b) You borrow $5 000 from your bank for 5 years to start a small clothing business in town. The bank charges interest at a fixed rate of 24% p.a. convertible monthly. There are two repayment options: Option 1- Monthly payments of interest only and one balloon

repayment of principal at the end of the loan term; and Option 2- Equal quarterly installments of principal and interest.

Required:

(i) Calculate the level quarterly installment that would amortize the loan over the 5-year term. **[4 marks]**

(ii) How much extra interest will you pay under Option 1 compared to Option 2 for the entire term of the loan? **[4 marks]**

(iii) Which of the two options would you take if you expect interest rates to increase in the future? Why? **[3 marks]**

(c) Osamaq Real Estate is a real estate company based in Bulawayo. The company is selling 4000 m^2 stands in the Burnside area of Bulawayo. The cash price for each serviced stand is $24 485. Alternatively, buyers may opt for a credit scheme that requires a minimum deposit of $15 000. However, the scheme price is $25 000, so that the balance of $10 000 is payable in 6 equal monthly installments of $1 666.67 each.

Required:

Calculate the effective annual cost of the credit from Osamaq, assuming that buyers take advantage of the full credit period. **[4 marks]**

(d) Given an effective rate of discount of 10.714% per annum, calculate:

(i) The effective annual rate of interest **[2 marks]**

(ii) The equivalent force of interest. **[2 marks]**

(iii) The quoted annual rate of interest convertible monthly **[2 marks]**

Total [25 marks]

END OF EXAMINATION PAPER