FACULTY OF COMMERCE DEPARTMENT OF FINANCE

BACHELOR OF COMMERCE HONOURS DEGREE IN FINANCE
PART I $1^{\text {ST }}$ SEMESTER FINAL EXAMINATION- DECEMBER 2015

FINANCIAL MATHEMATICS [CFI 1101]
TIME ALLOWED: 3 HOURS

## INSTRUCTIONS TO CANDIDATES

1. Attempt any FOUR (4) questions.
2. Show all workings.
3. Write neatly and legibly.

## INFORMATION FOR CANDIDATES

1. This paper contains FIVE (5) Questions.
2. Each full question carries a total of $\mathbf{2 5}$ marks and part marks are indicated in brackets at the end of each part question.
3. This paper contains SIX (6) printed pages, including the cover page.
4. A formula sheet is attached at the back of the question paper.

## QUESTION ONE

(a) You are considering the following three options for investing $\$ 10000$ for 5 years:

1. Interest of $8 \%$ per annum added once at the end of 5 years;
2. Interest of $6.98 \%$ per annum convertible every 3 months;
3. Interest of $7 \%$ per annum convertible every 6 months.

Which of the above options is the best? Show all workings.
(b) An investor deposited $\$ 1200$ into a savings account at Peace Bank on 1 January 2012. As at 31 December 2014 the balance in the investor's savings account was \$1 429.22. Interest rates are constant throughout the term of the deposit.

## Required:

(i) Calculate the annual effective rate of interest on the savings account up to 31 December 2014.
(ii) If the investor plans to withdraw the money as soon as the account balance reaches $\$ 2$ 400, what is the earliest date on which the investor can make the withdrawal? [NB: Assume that savings account withdrawals at Peace Bank are only processed on 30 June and on 31 December.]
(c) An investor is considering the following two options for investing \$200 000 for 90 days:

1. Treasury bills with 90 days remaining to maturity, par value of $\$ 1000$ and discount rate of $6 \%$ per annum;
2. A Negotiable Certificate of Deposit (NCD) with 90 days remaining to maturity, a price of \$200 000 and a maturity value of $\$ 203200$.

## Required:

(i) Calculate the fair price of each Treasury bill. [3]
(ii) How many Treasury bills can the investor buy at the price in (i) above? [1]
(iii) What is the annualised yield (interest rate) on the NCD? [2]
(iv) What is the equivalent yield for the Treasury bill? [2]
(v) Based merely on a comparison of the equivalent yield for the Treasury bill and the annualized yield on the NCD, which of the two options is better?

## QUESTION TWO

(a) As a student on attachment at Cashback Bank, you have developed an interest in foreign exchange markets. One day, you notice the following exchange rates on the bank's electronic board:

|  | Bid | Ask |
| :--- | :--- | :--- |
| ZAR/USD | 10.5000 | 10.6825 |
| BWP/USD | 9.3850 | 9.5481 |
| ZAR/BWP | 1.1482 | 1.1682 |

## Required:

(i) Based on the quotations above, demonstrate that a clever client could make a risk-free profit via triangular arbitrage.
(ii) How much risk-free profit will the client make based on an initial amount of ZAR1 million?
(iii) Calculate the ZAR/BWP maximum bid and minimum ask exchange rates that would eliminate the opportunity for risk-free profit.
(b) Assume that Zimbabwe introduces a new currency, the Zimbabwean Lifa (ZWL), on 1 January 2016. The Zimbabwean Lifa is quoted at 4.5000 against the South African Rand on 1 January 2016 (i.e. ZWL/ZAR=4.5000). As a firm believer in the Relative Purchasing Power Parity Theory (RPPP), you want to forecast the ZWL/ZAR exchange rate based on expected inflation differentials between South Africa and Zimbabwe for the next year. Expected inflation is $3.5 \%$ and $2.5 \%$ in Zimbabwe and South Africa respectively, while real interest rates in Zimbabwe and South Africa are $2.8 \%$ and $1.3 \%$ per annum respectively.

## Required:

(i) What do you expect the ZWL/ZAR exchange rate to be on 31 December 2016 according to the RPPP?
(ii) What do you expect the ZWL/ZAR exchange rate to be on 31 December 2016 according to the International Fisher Effect (IFE)?
(iii) Generally, what do you expect to happen to the value of the Zimbabwean Lifa relative to the South African Rand if inflation in Zimbabwe and South Africa is expected to remain constant for the next 5 years?
(c) Outline the relationship between Relative Purchasing Power Parity (RPPP), Generalized Fisher Effect (GFE), Interest Rate Parity (IRP), Rational Expectations Theory (RET), and the International Fisher Effect (IFE).

## QUESTION THREE

(a) Consider the following three Treasury bonds:

| Bond | Residual <br> Maturity <br> (years) | Coupon <br> rate (\%) | Yield to Maturity <br> $(\%)$ | Price | Duration <br> (years) | Convexity |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| A | 10 | 10 | 10 | $\mathbf{a}$ | 6.76 | b |
| B | 15 | 11 | 12 | $\mathbf{c}$ | d | 74.67 |
| C | 5 | 0 | e | 68.06 | $\mathbf{f}$ | 25.72 |

## Required:

Given that all Treasury bonds have a par value of $\$ 100$ and that coupons, if any, are paid annually, calculate the values of:
(i) a
(ii) b
(iii) C
(iv) d
(v) e
(vi) $f$
(b) An investor buys bond A in (a) above and holds it for 5 years. You are told that bond yields have remained constant over the investor's 5 -year holding period but the investor has only been able to reinvest coupons in a bank savings account earning interest at $8 \%$ per annum effective.

Calculate:
(i) The price at which the investor sells the bond.
(ii) The accumulated value of coupons. ..... [3]
(iii) The horizon date cash flow. ..... [1]
(iv) The investor's holding period yield. ..... [4]
Total [25]

## QUESTION FOUR

(a) Esther and Michael are a recently married young couple. Michael works as a manager at a local bank while Esther recently graduated with a degree in Finance from a local University. She is still unemployed but she expects to get a job one year from today paying a net salary of \$800 per month. Michael's net salary is $\$ 1200$ and both salaries are expected to remain constant for the next 10 years. The two have agreed that once Esther gets a job they will start saving $20 \%$ of their combined monthly income and deposit it on the last day of each month into a joint savings account earning interest at $12 \%$ per annum convertible monthly. Assume that both salaries are paid on the last day of each month in arrears.

## Required:

If Esther finds a job one year from today as expected;
(i) Calculate the balance in their joint savings account 5 years from today?
(ii) How many monthly deposits will it take for the balance in the joint savings account to reach $\$ 52$ 268.91?
(b) Jole has been appointed the new General Manager of a local insurance company. His new net salary will be $\$ 11779.72$ per month. Jole has just taken a 30-year mortgage loan for $\$ 300000$ at 12\% per annum effective. Repayments are made by level monthly instalments of principal and interest in arrears.

## Required:

(i) What percentage of his monthly salary will go towards the monthly instalment on the mortgage loan?
(ii) Construct an amortization schedule for the first 3 months of the loan repayment.
(iii) What proportion of the mortgage loan will be repaid between end of year 15 and end of year 20?
(iv) Calculate the total interest paid on the mortgage loan over the 30 -year term.

## QUESTION FIVE

(a) Consider a project that requires a once-off outlay of $\$ 300000$ and is expected to generate annual after-tax cash flows of $\$ 55000$ at the end of each year for the next 15 years. The company's cost of capital is $12 \%$.
Required:
(i) Calculate the Net Present Value (NPV) of the project.
(ii) Calculate the Internal Rate of Return (IRR) of the project.
(iii) Calculate the Modified Internal Rate of Return (MIRR) of the project.
(iv) Should the project be undertaken? Why?
(b) You recently took a 25-year mortgage loan for $\$ 80000$ at a fixed interest rate of $12 \%$ per annum effective. Repayments are scheduled to be made through monthly instalments of $\$ 801.18$ each for the first 10 years and monthly instalments of \$X each for the last 15 years of the loan term.

## Required:

(i) Given that the loan should be fully repaid at the end of the loan term, find the value of $X$.
(ii) If the bank allows you to continue with the level monthly instalments of $\$ 801.18$ each even after the first 10 years of the mortgage loan, how many years must be added to the original term of the loan in order for the loan to be fully repaid?

## END OF EXAMINATION PAPER

