# NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY FACULTY OF COMMERCE DEPARTMENT OF FINANCE <br> BACHELOR OF COMMERCE HONOURS DEGREE IN FINANCE <br> PART I - 2ND SEMESTER FINAL EXAMINATION - MAY 2006 <br> FINANCIAL MATHEMATICS II [CFI 1201] <br> TIME ALLOTTED: 3 HOURS 10 MINUTES <br> <br> INSTRUCTIONS 

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1. The paper is $\mathbf{3}$ hours $\mathbf{1 0}$ minutes, of which $\mathbf{1 0}$ minutes is reading time.
2. Attempt question 1 and 3 others.
3. Question 1 carries 40 marks.
4. Electronic calculators may be used.
5. Write legibly.

## Question One <br> [40 marks]

A bond has a fixed coupon rate of $24 \%$ p.a, a par value of $\$ 1000$ and a maturity date of December 1 2010. It pays coupons twice a year on June 1 and December 1.
1.1 Calculate the cum interest Dirty Price of the bond on 1 April 2006, if its yield to maturity is $28 \%$ p.a.
[8 marks]
1.2 Calculate the realized yield if immediately after settlement, on 1 April 2006, the yield to maturity falls to $26 \%$ p.a and remains at this level until the bond is disposed of, ex interest, on November 12009.
[8 marks]
1.3 Under what conditions will the realized yield at the Horizon date of an investor equal the promised yield to maturity at date of purchase?
[3 marks]
1.4 Calculate the Duration and convexity of the Bond on 1 April 2006.[12 marks]
1.5 Use the concept of Duration and Convexity to calculate Bond Price adjustment following the shift in yield to maturity in "1.2" above.
[6 marks]
1.6 Briefly, explain why Duration, alone, is not sufficient to explain, fully, the Bond price interest rate sensitivity calculated in " 1.5 " above
[3 marks]

## Question Two <br> [20 marks]

NUST Ltd, which has just gone through the launch stage of its business life cycle, is poised to experience phenomenal growth in earnings per share in the next 3 years, before competition catches up. After this period of super growth, earnings growth is expected to stabilize for the foreseeable future. At the end of the first year of the super growth stage, earnings per share, return on equity and payout ratio are projected to be $\$ 40$ 000, $50 \%$ and $20 \%$ respectively.

Thereafter, return on equity and payout ratio at the end of the first year of stabilized growth are expected to be $30 \%$ and $70 \%$ per year, respectively, for the foreseeable future.
2.1 Calculate the value of NUST Ltd stock if cost of equity is $30 \%$ during the super growth period and $25 \%$ during maturity stage of its Business life cycle.
[15 marks]
2.2 State any critical assumptions made in the model[s] used in "2.1" above.
[5 marks]

## Question Three

(20 marks)
The variance of the returns of a Portfolio of assets, $\sigma_{p}^{2}$, is given by:

$$
\sigma_{p}^{2}=\sum_{i=1}^{N} x_{i}^{2} \sigma_{i}^{2}+\sum_{\substack{i=1 \\ i \neq j}}^{N} \sum_{j=1}^{N} x_{i} x_{j} \sigma_{i j}
$$

where: $\sigma_{p}^{2}=\quad$ variance of the returns of a portfolio.
$x_{i}=\quad$ weight placed on the ith asset.
$x_{j}=\quad$ weight placed on the $\boldsymbol{j} t \boldsymbol{h}$ asset.
$\sigma_{i}^{2}=\quad$ variance of the returns of the $\boldsymbol{i t h}$ asset.
$N \quad=\quad$ Number of assets in the portfolio.
$\sigma_{i j}=\quad$ covariance between the returns of the $\boldsymbol{i t h}$ and the $\boldsymbol{j} t \boldsymbol{h}$ asset.
3.1 Show that the relevant risk, when an asset (sayk) is held in a well diversified portfolio, is the sum of the covariance of the asset with other assets in the portfolio.
[7 marks]
3.2 Suggest and justify a Capital Asset Pricing Model [CAPM] based strategy for an investment that replicates the expected return of any other asset. [6 marks]
3.3 What are the critical assumptions of the Capital Asset Pricing Model?.
[7 marks]

## Question Four

[20 marks]
The stock of Delta Ltd, which is currently trading at \$200 000 per share, has an annual volatility (SD) of $40 \%$.

You expect to receive a substantial amount of cash at the end of 6 months from now, which cash your intend to use to buy Delta stock. A dealer offers you the option, for a price, to buy Delta stock at $\$ 250000$ six months from now.

What is the fair value, using a 2 -step Binominal process, of the option if riskfree rate of return is $20 \%$ p.a?
[20 marks]

## Question Five

## (20 marks)

5.1 State and explain the critical assumptions of Parity conditions in money and capital markets.
[8 marks]
5.2 Starting with the absolute version of the Purchasing Power Parity Theory [PPPT] prove the relative version of Purchasing Power Parity Theory [PPPT] i.e. that

$$
\frac{S_{t}-S_{t-1}}{S_{t}}=\frac{i_{h}-i_{f}}{1+i_{f}}
$$

## Where:

$S_{t} \quad=\quad$ Exchange rate in period $t$
$S_{t-1}=\quad$ Exchange rate in period $t-1$
$i_{h}=\quad$ Inflation rate in the domestic [home] currency.
$i_{f} \quad=\quad$ Inflation rate in the foreign currency.
[12 marks]

