## NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY FACULTY OF COMMERCE DEPARTMENT OF FINANCE BACHELOR OF COMMERCE HONOURS DEGREE IN FINANCE PART I- $2^{\text {ND }}$ SEMESTER FINAL EXAMINATION - JUNE 2007 <br> FINANCIAL MATHEMATICS II CFI 1201 <br> TIME ALLOTTED: 3 HOURS 10 MINUTES

## INSTRUCTIONS

1. The paper is $\mathbf{3}$ hours $\mathbf{1 0}$ minutes, of which $\mathbf{1 0}$ minutes is reading time.
2. Attempt question 1 and 3 others.
3. Question 1 carries $\mathbf{4 0}$ marks.
4. Electronic calculators may be used
5. Write legibly.

## Question One

[40 marks]

NUST Pension Fund has a One-off obligation of \$14000000, due at the end of 3 years. It has decided to discharge the obligation by purchasing, today, a portfolio of government bonds. The yield curve is flat at 42\%.
1.1 Calculate the present value of the obligation.
[3 marks]
1.2 What is the duration and convexity of the future obligation? [3 marks]
1.3 Suggest a strategy for immunizing bond portfolio, purchased to discharge the obligation, against interest rate risk in times of high interest rate volatility. Explain fully
[7 marks]
1.4 Determine the reaction of Portfolio value to 50 basis point a parallel rise, immediately after purchase, in the yield curve.
[8 marks]
1.5 Suppose a bond pays an annual coupon of $35 \%$ on a par value of $\$ 1000$. Calculate the realized yield following the 50 basis points, and only, rise in yield to maturity, if maturity of the bond is 4 years and horizon date of the investor is 3 years.
[19 marks]
Question Two
[20 marks]
Consider two assets, A and B , with performance measures tabled below:

| ASSET | A | B |
| :--- | :--- | :--- |
| $\bar{R}$ | $30 \%$ | $20 \%$ |
| $\sigma$ | $25 \%$ | $18 \%$ |

2.1 What are the weights and expected return of a fully hedged portfolio of assets $\mathbf{A}$ and $\mathbf{B}$, if the correlation coefficient between their returns is $\mathbf{- 1}$ ?
[10 marks]
2.2 What are the weights and expected return of the minimum variance portfolio if the correlation coefficient between their returns is zero. [10 marks]

## Question Three

## [20 marks]

You are considering two investment strategies over your horizon date of 3 years. The first is to purchase a two year maturity, government bond, whose spot yield is $28 \%$, and roll forward the investment at the end of the second year.

The second strategy is to purchase a government bond with a maturity of 3 years (your horizon date). The spot yield to maturity on this bond is $30 \%$.
3.1 What is the observed one year forward rate at the beginning of year 3?
[7 marks]
3.2 Suppose the expected one year spot rate at the beginning of year 3, is $36 \%$, which strategy is more profitable? Explain
[6 marks]
3.3 What is the yield curve implication of the decision taken in 3.2 above?
[7 marks]

## Question Four [20 marks]

Delta, whose stock is currently trading at $\$ 20000$, has proposed an interim dividend of $\$ 1500$ per share payable in 91 days. The yield on 91 day Treasury Bills is $100 \%$ per annum. You have been offered to buy or sell, the stock, 91 days forward at a forward price of $\$ 23000$.
4.1 What position should you take, today and at delivery date to profit?
[8 marks]
4.2 Determine, and explain the nature of the profit if the spot price at delivery date is $\$ 18000$.
[8 marks]
4.3 State the critical assumptions you have made.
[4 marks]

## Question Five

[20 marks]

You have been offered the option to purchase a piece of land for $\$ 40000000$ which is its current price, 6 months from now, when you come back from a working visit to the diaspora.
5.1 What is the fair price, using two step binomial process, of the option, if the yield on 182 day Treasury bill is $60 \%$ per annum. and land price volatility is 80\% per annum.
[5 marks]
5.2 Suppose you have the option to exercise early and the land does not have rentable income before the expiry date, is it ever profitable to do so? Explain
[15 marks]

