

**NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY**  
**FACULTY OF COMMERCE**  
**DEPARTMENT OF FINANCE**  
**B.COMM (HONOURS) DEGREE IN FINANCE**  
**PART I – FINAL EXAMINATIONS – JUNE 2010**  
**FINANCIAL MATHEMATICS II [CFI 1201]**  
**TIME ALLOWED: 3 HOURS**

**Instructions to Candidates**

1. Attempt TWO (2) Questions from SECTION A
2. Attempt ANY THREE (3) Questions from SECTION B
3. To Obtain full marks Show ALL appropriate steps to your answers
4. Carefully read and adhere to the instructions on the provided answer booklet

**Requirements :**

**A non-programmable Scientific calculator**

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**SECTION A [COMPULSORY – 40 MARKS]**

**QUESTION ONE**

- (a) Define the following terms as used in financial mathematics:
- (i) Reinvestment risk [1 mark]
  - (ii) Arbitrage [2 marks]
- (b) Define a fixed income bond and outline its main features [3 marks]
- (c) With the aid of a labeled diagram demonstrate how an investor may achieve efficient diversification [3 marks]
- (d) List the criticisms of the Capital Asset Pricing Model (CAPM). [3 marks]
- (e) (i) Yield to maturity (YTM) on bond is expected to move from 9.4% to 10.2%. If the bond has a 4 year duration (Macaulay duration or discounted mean term (DMT)), what is the expected percentage price change. [2 marks]
- Given the following information about market portfolio (m) and a bond in the market (i):
- $\sigma_i = 8\%$ ,  $\sigma_{im} = 11\%$ ,  $\sigma_m = 7\%$ ,  $D_m = 8.5$  years and  $D_i = 6.08$  years
- (iii) Calculate the Beta for the Bond ( $\beta_i$ ) and interpret your result. [2 marks]

(f) Consider a large Portfolio with N Securities in which

- (i) Equal amounts are invested in each asset
- (ii) Security returns are not correlated
- (iii) Securities have the same risk.

Show how diversification can eliminate individual security risk and highlight the importance of this result to investors. [3 marks]

**[Total 20 Marks]**

### QUESTION TWO

(a) A Government loan stock has the following features:

- Face value = 1000
- Coupon = 20%
- Maturity date = Dec 31 2010
- Coupon dates = June 30 and Dec 31

You purchase the loan stock cum interest on April 18, 2006 when yield to maturity is 24% and sell it ex interest on December 10 2009 when yield to maturity is 18%

**Calculate:**

- (i) Dirty price of bond at purchase [5 marks]
- (ii) Dirty price of bond at sale [6 marks]
- (iii) Realized yield for the bond investment. [5 marks]

(b) Given the equation  $R_i = \alpha + B_i R_m + \varepsilon_i$  for the return on a security in the market, clearly define the terms on the right hand side (RHS) of the equation and demonstrate how the total risk of the security can be decomposed its non market component  $(\sigma_{\varepsilon_i}^2)$  and market component  $B_i^2 \sigma_m^2$  [5 marks]

**[Total 20 Marks]**

**SECTION B – ANSWER ANY THREE QUESTIONS [60 MARKS]**

**QUESTION THREE**

A pension fund expects to make payments of \$100,000 per annum at the end of each of the next five years. It wishes to immunise these liabilities by investing in two zero coupon bonds A and B which mature in five years and in one year respectively. The rate of interest is 5% per annum effective.

- (a)
- (i) Show that the present value of the liabilities is \$432,948. [3 marks]
  - (ii) Show that the duration of the liabilities is 2.9 years. [4 marks]
- (b) Calculate the nominal amounts of the two zero coupon bonds which must be invested in bonds A and B if the pension fund is to equate the present value and duration of assets and liabilities. [5 marks]
- (c)
- (i) Calculate the convexity of the assets. [4 marks]
  - (ii) Without calculating the convexity of the liabilities, comment on whether you think the conditions for immunisation have been achieved. [2 marks]

Using your results in (c) (i) and (ii) above, outline how an investor can arbitrage with convexity. [2 marks]

**[Total 20 Marks]**

**QUESTION FOUR**

- (a) Given the following table for the spot rate and the time to maturity (TTM) for Bonds on the market.

Spot Rate	TTM
20%	1
30%	2
35%	3

Calculate the following:

- (i) The implied forward rate for 1 year at the beginning of year 1. [1 mark]
  - (ii) The implied forward rate for 2 years at the beginning of year 2. [2 marks]
  - (iii) The 1-year forward rate at the beginning of year 3. [2 marks]
- (b) Suppose your horizon date is 2 years and you expect the future rate for 1 year at the beginning of year 2 to be 45%, what is the implication for term structure of interest rates in (i) above? [3 marks]

(c)

(i) With the aid of examples, describe the main characteristics of capital market instruments [4 marks]

(ii) Faithful Chow, founder and president of Sea Wind Trading, is wondering whether the company should make its first public sale of common stock, and if so, at what price. The company's financial plan envisages rapid growth over the next 4 years but only moderate growth afterwards. Forecast earnings and dividends are as follows:

Year:	1	2	3	4	5	6	7	8
Earnings per share	\$2.45	3.11	3.78	4.00	4% growth thereafter			
Dividends per share	\$1.00	1.20	1.44	1.73	4% growth thereafter			

What is the fair price for the stock if investors require a return of 10% and a price-earnings ratio of 8, comment on your result. [8 marks]

**[Total 20 Marks]**

#### QUESTION FIVE

(i) Given three assets A, B, C and D in the table below; compute the variables indicated by the question marks.

Outcome	Probability	R <sub>A</sub>	R <sub>B</sub>	R <sub>C</sub>	R <sub>D</sub>
1	.25	24%	28%	4%	10%
2	.50	12%	12%	8%	0%
3	.25	0%	4%	28%	-10%
$\bar{R}$		12	14	12	0%
$\sigma^2$		72	76	88	? [1]
$\sigma$		8.49	8.72	9.38	? [2]
CVaR		0.7075			? [1]
X <sub>i</sub>		35%	20%	45%	0%
$\sigma_{AD}$	? [3]				
$\sigma_{CB}$	? [2]				
$\rho_{AC}$	? [3]				
E <sub>P</sub>	? [1]				
$\sigma_P$	? [3]				

- (ii) Define the term marginal portfolio variance and state its value for asset D justifying your answer. [4 marks]

**[Total 20 Marks]**

**QUESTION SIX**

- (i) Distinguish between a forward and a futures contract [4 marks]  
(ii) Using an ideal arrangement of a futures contract illustrate the important role played by the clearing house in futures trading. [6 marks]  
(iii) Given that the Purchasing Power Parity (PPP) relationship is given by:

$$PPP = \frac{e_t}{e_0} = \frac{(1+i_h)^t}{(1+i_f)^t} \approx i_h - i_f$$

Where  $e_t$  = future spot rate  
 $e_0$  = spot rate  
 $i_h$  = home inflation  
 $i_f$  = foreign inflation  
 $t$  = the time period

and that the Fisher Effect (FE) is given by:

$$FE = r_h - r_f = i_h - i_f$$

where  $r_h$  = home nominal rate  
 $r_f$  = foreign nominal rate

- (i) State the conditions (assumptions) under which the above formulae are valid. [4 marks]  
(ii) Given the above relationships and use general reasoning or mathematical arguments to show that if the relationship for the International Fisher Effect (IFE) is  $IFE = PPP + FE$ , then the formulae:

$$IFE = \frac{\bar{e}_t}{e_0} = \frac{(1+r_h)^t}{(1+r_f)^t} \text{ is true} \quad [6 \text{ marks}]$$

**[Total 20 Marks]**

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\*\*\*END OF EXAMINATION\*\*\*