



**National University of
Science and Technology**
Think in Other Terms



FACULTY OF COMMERCE
DEPARTMENT OF FINANCE
BACHELOR OF COMMERCE HONOURS DEGREE IN
FINANCE
PART I 1ST SEMESTER FINAL EXAMINATION - MAY 2011
FINANCIAL MATHEMATICS II [CFI 1201]
TIME ALLOWED: 3HOURS

INSTRUCTIONS TO CANDIDATES

1. Answer **All** Questions in **Section A** and any **TWO** from Section B
2. Start the answer to each full question on a fresh page of the answer sheet.
3. Questions may be written in any order, but must be legibly numbered.
4. Write legibly, showing all workings.

INFORMATION TO CANDIDATES

1. Section A carries a total of 60 marks and each question in Section B carries a total of 20 marks.
2. The businesses in this question paper are intended to be fictitious
3. The paper contains **FOUR (4) QUESTIONS**.

SECTION A (COMPULSORY)

Question One [60 marks]

(a) An investor purchases a \$1000, 10% coupon bond at a price yielding 11% p.a over 20 years. Coupons are payable annually.

Calculate:

- (i) The equilibrium price of the bond. [4 marks]
- (ii) The investor's realized yield if yields increase to 12% at the end of 10 years and the investor holds the bond to maturity. [4 marks]
- (iii) The investor's holding period yield if yields increase to 12% at the end of 10 years and the investor sells the bond at the end of 17 years. [5 marks]

(b) What can be said about the Macaulay duration of the bond given your results in (a)(iii) above? [3 marks]

(c) An analyst has estimated the following return distributions for stocks A and B:

State of the economy	Probability	Return on A (%)	Return on B (%)
Boom	0.3	20	7
Normal	0.6	15	16
Slump	0.1	5	22

Using the information in the table above, determine:

- (i) The expected return and standard deviation of A and B. [4;6 marks]
- (ii) The covariance of returns for A and B. [3 marks]
- (iii) The correlation coefficient of returns of A and B. [2 marks]
- (iv) The expected return and standard deviation of a portfolio invested 40% in A and 60% in B. [2;3 marks]
- (v) The minimum variance portfolio comprising stocks A and B. [4 marks]
- (vi) The optimal portfolio of risky assets given that the risk free rate of return is 10%. [5 marks]

- (vii) The complete portfolio consisting of the optimal portfolio of risky assets and the risk free asset, given that the investor's risk aversion factor is 4. [6 marks]
- (viii) The expected return and standard deviation of the complete portfolio.[2;3 marks]
- (ix) The equation of the capital asset line (CAL) for A and B [4 marks]

Total [60 marks]

SECTION B

Question Two

(a) Define the following risks associated with bonds:

- (i) Interest rate risk [2 marks]
- (ii) Reinvestment rate risk [2 marks]

(b) Consider a \$1000, 10% coupon bond that is issued at a price of \$962 and has a term to maturity of 5 years, with coupons payable annually.

Calculate:

- (i) The yield to maturity of the bond [3 marks]
- (ii) The Macaulay duration of the bond [5 marks]
- (iii) The Convexity of the bond [5 marks]
- (iv) The approximate change in the value of the bond following a decrease in yields to 10.5% [3 marks]

Total [20 marks]

Question Three

(a) Shawntel Ltd is a public company that has just paid a dividend of \$2.00 per share. The company's shares are currently trading at \$46 and dividends are expected to grow at a constant rate of 5.4% indefinitely. The return on equity (ROE) for the company is 15%.

Calculate:

- (i) The implied required return on equity (k_e) for Shawntel Ltd. [2 marks]
- (ii) The implied payout ratio [2 marks]

- (iii) The value per share if investors believe that the dividend growth rate will be 7% p.a for the first four years and 5.4% thereafter. (NB: Assume the same k_e as in (i) above) [5 marks]
- (b) Distinguish between net operating cash flow and free cash flow to equity. [2 marks]
- (c) The yield on 4-year treasury bonds is 15% and that on 3-year Treasury notes is 12%. Determine the implied 1-year forward rate 3 years from now. [3 marks]
- (d) Briefly explain the following term structure theories:
- (i) The Liquidity Preference Theory [2 marks]
 - (ii) The Segmentation Theory [2 marks]
 - (iii) The Rational Expectations Theory [2 marks]

Total [20 marks]

Question Four

- (a) State any **FOUR** assumptions underlying Modern Portfolio Theory (MPT). [4 marks]
- (b) Determine whether each of the following portfolios is efficient given the following equation for the Capital Market Line (CML): $R_p = 0.06 + 0.6\sigma_p$ [8 marks]

Portfolio	Expected Return (%)	Standard Deviation (%)
A	8	5
B	9.9	6.5
C	9	5
D	11	8

- (c) Interest rates are 8% and 5% in South Africa and in the US respectively. According to the International Fischer Effect (IFE), what is the expected depreciation in the South African rand? [3 marks]
- (d) Show that the condition: $\frac{i_h - i_f}{1 + i_f} = \frac{r_h - r_f}{1 + r_f}$ is consistent with the IFE; where i_h and i_f are home and foreign inflation rates respectively and r_h and r_f are home and foreign nominal interest rates respectively. [6 marks]

Total [20 marks]

END OF PAPER