

FACULTY OF COMMERCE DEPARTMENT OF FINANCE
BACHELOR OF COMMERCE HONOURS DEGREE IN FINANCE
PART I 2ND SEMESTER SUPPLEMENTARY EXAMINATION - AUGUST 2011 FINANCIAL MATHEMATICS II [CFI 1201]

TIME ALLOWED: 3HOURS

## INSTRUCTIONS TO CANDIDATES

1. Attempt any FOUR questions
2. Start the answer to each full question on a fresh page of the answer sheet.
3. Questions may be written in any order, but must be legibly numbered.
4. Write legibly, showing all workings.

## INFORMATION TO CANDIDATES

1. Each Question carries a total of $\mathbf{2 5}$ marks.
2. The businesses in this question paper are intended to be fictitious
3. The paper contains FOUR (4) QUESTIONS.

## Question One

(a) An investor purchases a $\$ 1000,10 \%$ coupon bond at a price yielding $11 \%$ p.a over 20 years. Coupons are payable annually.
Calculate:
(i) The equilibrium price of the bond.
(ii) The investor's realized yield if yields increase to $12 \%$ at the end of 10 years and the investor holds the bond to maturity.
[5 marks]
(iii) The investor's holding period yield if yields increase to $12 \%$ at the end of 10 years and the investor sells the bond at the end of 17 years. [5 marks]
(b) Given results in (ii) and (iii) of (a) above, determine, giving reasons, whether the bond duration is below or above 17 years.
[6 marks]
(c) Discuss any TWO weaknesses of duration analysis as an approach to bond risk analysis.
[4 marks]
Total [25 marks]

## Question Two

An analyst has estimated the following return distributions for stocks A and B:

| State of the <br> economy | Probability | Return on A (\%) | Return on B (\%) |
| :--- | :--- | :--- | :--- |
| Boom | 0.3 | 20 | 7 |
| Normal | 0.6 | 15 | 16 |
| Slump | 0.1 | 5 | 22 |

Using the information in the table above, determine:
(i) The expected return and standard deviation of A and B.
[4;6 marks]
(ii) The covariance of returns for A and B . [3 marks]
(iii) The correlation coefficient of returns of $A$ and $B$.
[2 marks]
(iv) The expected return and standard deviation of a portfolio invested $40 \%$ in A and $60 \%$ in B.
(v) The minimum variance portfolio comprising stocks A and B. [5 marks]

Total [25 marks]

## Question Three

(a) Define the following risks associated with bonds:
(i) Interest rate risk [4 marks]
(ii) Reinvestment rate risk
(b) Consider a $\$ 1000,10 \%$ coupon bond that is issued at a price of $\$ 962$ and has a term to maturity of 5 years, with coupons payable annually.

Calculate:
(i) The yield to maturity of the bond
[4 marks]
(ii) The Macaulay duration of the bond
[6 marks]
(iii) The Convexity of the bond
[5 marks]
(iv) The approximate change in the value of the bond following a decrease in yields to 10.5\%
[3 marks]
Total [25 marks]

## Question Four

(a) Shawntel Ltd is a public company that has just paid a dividend of $\$ 2.00$ per share. The company's shares are currently trading at $\$ 46$ and dividends are expected to grow at a constant rate of $5.4 \%$ indefinitely. The return on equity (ROE) for the company is $15 \%$. Calculate:
(i) The implied required return on equity $\left(\mathrm{k}_{\mathrm{e}}\right)$ for Shawntel Ltd. [3 marks]
(ii) The implied payout ratio
(iii) The value per share if investors believe that the dividend growth rate will be $7 \%$ p.a for the first four years and $5.4 \%$ thereafter. (NB: Assume the same $k_{e}$ as in (i) above)
(b) Distinguish between net operating cash flow and free cash flow to equity. [2 marks]
(c) The yield on 4-year treasury bonds is $15 \%$ and that on 3-year Treasury notes is $12 \%$. Determine the implied 1-year forward rate 3 years from now.
(d) Briefly explain the following term structure theories:
(i) The Liquidity Preference Theory [3 marks]
(ii) The Segmentation Theory [3 marks]
(iii) The Rational Expectations Theory

## Question Five

(e) State any FOUR assumptions underlying Modern Portfolio Theory (MPT). [4 marks]
(f) Determine whether each of the following portfolios is efficient given the following equation for the Capital Market Line (CML): $R_{p}=0.06+0.6 \sigma_{p}$
[8 marks]

| Portfolio | Expected Return <br> $(\%)$ | Standard Deviation <br> $(\%)$ |
| :--- | :--- | :--- |
| A | 8 | 5 |
| B | 9.9 | 6.5 |
| C | 9 | 5 |
| D | 11 | 8 |

(g) Interest rates are $8 \%$ and 5\% in South Africa and in the US respectively. According to the International Fischer Effect (IFE), what is the expected depreciation in the South African rand? [3 marks]
(h) Show that the condition: $\frac{i_{h-i_{f}}}{1+i_{f}}=\frac{r_{h-r_{f}}}{1+r_{f}}$ is consistent with the IFE; where $i_{h}$ and $i_{f}$ are home and foreign inflation rates respectively and $r_{h}$ and $r_{f}$ are home and foreign nominal interest rates respectively.

Total [20 marks]

## END OF EXAMINATION PAPER

