



**National University of  
Science and Technology**  
Think in Other Terms



FACULTY OF COMMERCE  
DEPARTMENT OF FINANCE  
BACHELOR OF COMMERCE HONOURS DEGREE IN FINANCE  
PART I 2<sup>nd</sup> SEMESTER FINAL EXAMINATION– MAY 2012  
FINANCIAL MATHEMATICS II [CFI 1201]  
TIME ALLOWED: 3 HOURS

**INSTRUCTIONS TO CANDIDATES**

1. Answer **QUESTION ONE** and any **THREE (3)** other questions.8
2. Show all workings.
3. Write neatly and legibly.

**INFORMATION TO CANDIDATES**

1. This paper contains **FIVE (5)** Questions.
2. **Question One** carries a total of **40 marks** and **Questions 2-5** carry a total of **20 marks** each.
3. This paper contains **FIVE (5)** printed pages.
4. Candidates may write on the question paper but shall not write in the answer booklet during reading time.
5. The businesses in this question paper are intended to be fictitious.

## **QUESTION ONE (COMPULSORY)-40 MARKS**

### **Question 1.1**

- a) A 15-year callable treasury bond is currently priced at \$84 and a corresponding 15-year puttable Treasury bond has an embedded put option value of \$2.50. The 15-year option-free Treasury bond is currently priced at \$87.20. Calculate:
- The embedded call option value for the bond [2 marks]
  - The current price of the puttable Treasury bond [2 marks]
  - The strike price of the bond if the time remaining to maturity is 6 months and the continuously compounded risk free rate of interest is 5% per annum.[4 marks]
- b) Consider a portfolio that offers an expected return of 12% and has a standard deviation of 18%. Treasury bills offer a 7% rate of return. What is the maximum level of risk aversion for which the risky portfolio is still preferred to bills? [3 marks]

### **Question 1.2**

Assume a world with only two risky assets X and Y, with expected returns of 10.9% and 11.5% respectively, and standard deviations of 2.21% and 3.5% respectively. The correlation coefficient of returns between X and Y is -0.1744. Calculate:

- The weights of X and Y in the minimum variance portfolio. [3 marks]
- The expected return and standard deviation of the minimum variance portfolio. [2;3 marks]
- The weights of X and Y in the optimal portfolio for an investor with a risk aversion (A) of 7. [3 marks]
- The expected return and standard deviation of the optimal portfolio in (iii) above. [2;3 marks]
- The utility scores of the minimum variance portfolio and the optimal portfolio and comment on the results. [2;2;2 marks]

### Question 1.3

- a) State any **TWO** weaknesses of the constant growth dividend discount model.[2 marks]
- b) State Redington's conditions for immunization of a portfolio consisting of both assets and liabilities. [3 marks]
- c) Given a modified duration (volatility) of 3.5, what is the approximate percentage change in the price of a bond given a 0.5% increase in the yield? [2 marks]
- TOTAL [40 marks]**

### QUESTION TWO

You determine that the optimal risky portfolio is invested 25% in asset A and 75% in asset B. The expected rate of return of the optimal risky portfolio is 18% and the standard deviation is 28%. The risk free rate of return is 8%.

- a) A client chooses to invest 70% of his funds in the optimal risky portfolio and 30% in the risk free asset. Calculate:
- i. The expected return and standard deviation of the client's portfolio. [2;2 marks]
  - ii. The client's risk aversion (A). [3 marks]
  - iii. The investment proportions of A and B in the client's portfolio if the optimal risky portfolio is invested? [3 marks]
- b) What is the reward-to-variability ratio (Sharpe ratio) of:
- i. The optimal risky portfolio? [2 marks]
  - ii. The client's portfolio? [2 marks]
- c) Suppose the client decides to invest a proportion  $y$  of his total investment budget in the optimal risky portfolio so that the complete portfolio will have an expected return of 16%. Calculate:
- i. The proportion  $y$ . [2 marks]
  - ii. The standard deviation of the complete portfolio. [1 marks]
- d) Suppose instead that the client decides to maximize expected return subject to the constraint that the standard deviation of the complete portfolio does not exceed 18%. Calculate:
- i. The proportion  $y$  invested in the optimal risky portfolio. [1 mark]
  - ii. The expected return of the complete portfolio. [2 marks]
- TOTAL [20 marks]**

### **QUESTION THREE**

- a) Assume a world with only two risky assets A and B, with expected returns of 10.9% and 11.5% respectively, and standard deviations of 2.21% and 3.5% respectively. The correlation coefficient of returns between A and B is -1. What proportions would need to be invested in A and B in order to eliminate risk? [3 marks]
- b) An on-the-money 6-month call option on a share of Z Ltd stock with a strike price of \$4 is currently valued at \$0.25. The continuously compounded risk free rate of interest is 5% per annum. Calculate the fair price of a put option on the same share with the same strike price and maturity. [3 marks]
- c) Draw the profit diagrams for the long call and long put options in (b) above. [4 marks]
- d) Smith has \$400 available for investment. He has three alternative ways of investing the money. Firstly, he can invest all the money in 100 shares of Z Ltd at the current price. Secondly he can buy 1600 Z Ltd stock call options at the current price of \$0.25 and thirdly, he can buy 100 Z Ltd stock options and invest the balance of \$375 at the risk free rate of interest for 6 months. Calculate:
- The returns on the three alternative portfolios if the share price after 6 months is \$4.50. [4 marks]
  - The returns on the three alternative portfolios if the share price after 6 months is \$3.50. [4 marks]
  - Comment on the above results. [2 marks]

**TOTAL [20 marks]**

### **QUESTION FOUR**

- a) State the mean-variance rule for portfolio selection. [2 marks]
- b) Why is the minimum variance portfolio important in portfolio selection? [3 marks]
- c) What is meant by a fully-hedged portfolio? [2 marks]
- d) Define investor risk aversion. [3 marks]
- e) Draw the payoff diagram for: i) a protective put and ii) a covered call. [3;3 marks]
- f) Asset X and asset Y have standard deviations of returns of 8% and 10% respectively and a covariance of returns of -72. Calculate the correlation coefficient of returns between X and Y. [2 marks]

- g) Given that the standard deviation of returns on the market portfolio is 5% and that the covariance of returns between an equally-weighted portfolio of X and Y and the market portfolio is 40, calculate the beta of the equally-weighted portfolio. [2 marks]

**TOTAL [20 marks]**

### **QUESTION FIVE**

- a) XYZ Ltd has just paid a dividend of \$2 to its common shareholders. The dividend is expected to grow at a constant rate of 4% per annum indefinitely and the required return on equity is expected to remain at the current rate of 7%. Calculate the fair value of the XYZ share. [3 marks]
- b) Suppose instead that analysts forecast a dividend growth rate of 8% per annum for the first three years before leveling off at a constant rate of 4% p.a indefinitely. The required return of equity is not expected to change however. Calculate the fair value of the share. [4 marks]
- c) You are given that the ABC share has a beta of 1.5 and the expected market risk premium is 3%. The risk free rate is 4%. Calculate the equilibrium expected rate of return on the ABC share. [3 marks]
- d) If the expected return on the ABC share is 8%, determine whether the share is overvalued or undervalued. [2 marks]
- e) Calculate the total risk of the ABC share returns if the standard deviation of market returns is 6% and the security specific risk of ABC shares is 4%. [3 marks]
- f) Plot the ABC share on the Security Market Line (SML). [3 marks]
- g) State any **TWO** differences between the Capital Market Line (CML) and the SML. [2 marks]

**TOTAL [20 marks]**

**END OF EXAMINATION PAPER**