National University of Science and Technology
Think in Other Terms

FACULTY OF COMMERCE

DEPARTMENT OF FINANCE

BACHELOR OF COMMERCE HONOURS DEGREE IN FINANCE

PART IV FINAL EXAMINATION- DECEMBER 2014

ADVANCED ASSET PRICING THEORY AND PRACTICE [CFI 4101]

TIME ALLOWED: THREE (3) HOURS

INSTRUCTIONS TO CANDIDATES

1. **Question One is Compulsory**
2. Attempt Question One and any other **THREE (3)** questions.
3. All workings to be shown.
4. Write legibly.
5. All attempted questions should be clearly numbered.

INFORMATION TO CANDIDATES

1. All question carry equal marks [25 marks]
2. Marks for each sub-question are shown in square brackets [ ]
3. Questions may be written in any order
4. You may use electronic calculators.
5. This examination paper consists of 5 printed pages; including the cover page

Question One [Compulsory]

i. Explain the following terms in the context of portfolio theory.

a. Feasible set **[2 marks]**

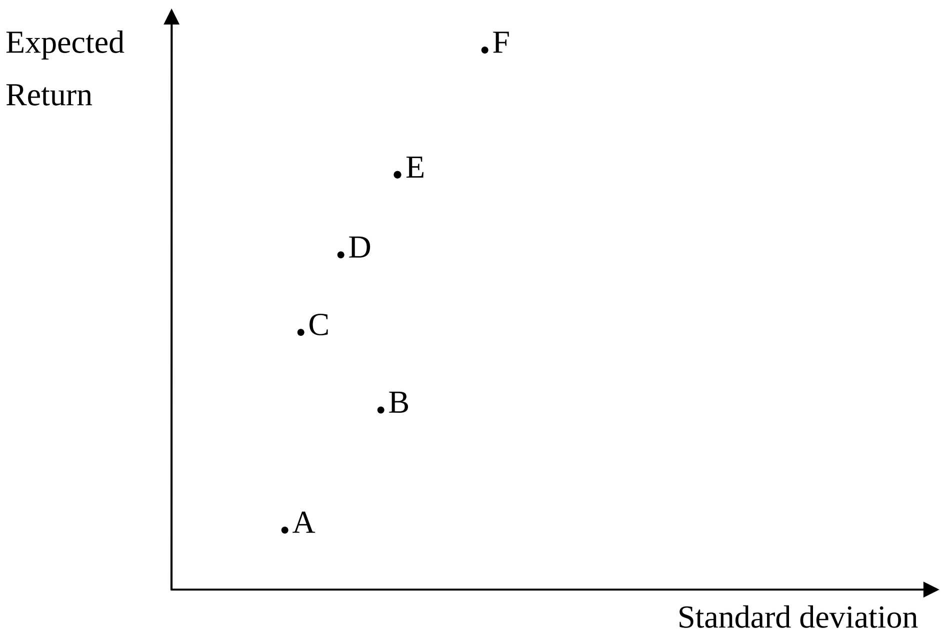
b. Efficient frontier **[2 marks]**

c. The market portfolio **[2 marks]**

d. The beta coefficient of a stock **[2 marks]**

e. The security market line **[2 marks]**

ii. Which portfolios among A, B, C and D pictured below could be on an efficient frontier? Explain your answer in detail. **[2 marks]**



iii. The variance of a portfolio of securities σ_p^2 is given by the formula;

$$\sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \rho_{ij} \sigma_i \sigma_j$$

where $x_i =$ proportion of total investment in Security i

$\rho_{ij} =$ correlation coefficient between Security i and Security j

Show that, for a two-security minimum variance portfolio, the proportions invested in stocks A and B are:

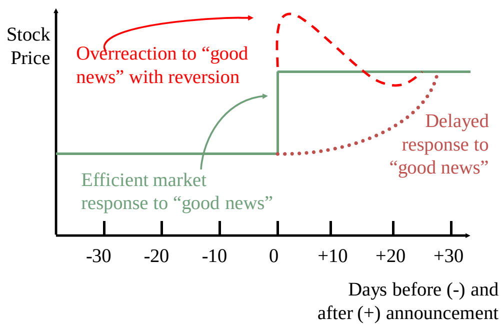
$$x_A = \frac{\sigma_B^2 - \sigma_A \sigma_B \rho_{AB}}{\sigma_A^2 + \sigma_B^2 - 2\sigma_A \sigma_B \rho_{AB}}$$ **[13 marks]**

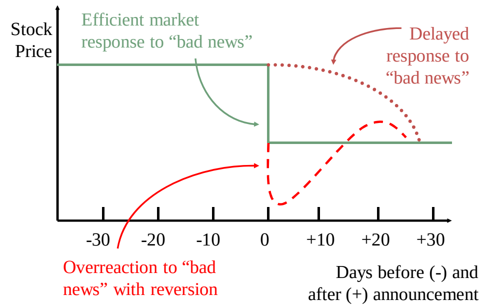
$$x_B = 1 - x_A$$

Question Two

i. Define the concept of efficiency in capital markets clearly distinguishing between the various forms of efficiency. **[6 marks]**

ii. Using the two graphs shown below, and any other details, explain the significance and impact of 'window period' selection in testing for market efficiency (clearly outline the advantages and disadvantages of a narrow or wide window period). **[8 marks]**





iii. Tests of market efficiency are often referred to as joint tests of two hypotheses - the hypothesis that the market is efficient and an expected returns model. Explain why this is often the case. **[4 marks]**

iv. Is it ever possible to test market efficiency alone? (i.e. without jointly testing an asset pricing model). **[3 marks]**

v. One explanation of the turn-of-the-year or January effect has to do with sales and purchases related to the tax year. However, studies have shown that the January effect occurs internationally, even in countries where the tax year does not start in January. Speculate on a good reason for this. **[4 marks]**

Question Three

i. *"Market portfolio at the heart of the model is theoretically and empirically elusive. It is not theoretically clear which assets (e.g., human capital) can legitimately be excluded from the market portfolio, and data availability substantially limits the assets that are included. As a result, tests of CAPM are forced to use compromised proxies for market portfolio, in effect testing whether the proxies are on the min-variance frontier."* Fama & French (2004).
Examine the impact and/or relevance of the above assertion in testing for the validity of the CAPM. **[5 marks]**

ii. The return , $E(\tilde{R}_P)$, and the standard deviation, ,$\sigma(\tilde{R}_P)$, from a portfolio consisting of the market portfolio and the a risky security *i*, can be given as;

$$E(\tilde{R}_P) = w_i E(\tilde{R}_i) + (1 - w_i)E(\tilde{R}_m)$$

$$\sigma(\tilde{R}_P) = [w_i^2 \sigma_i^2 + (1 - w_i)^2 \sigma_m^2 + 2w_i(1 - w_i)\sigma_{im}]^{\frac{1}{2}}$$

Show that $$\left.\frac{\partial \sigma(\tilde{R}_P)}{\partial a}\right|_{w_i=0} = \frac{\sigma_{im} - \sigma_m^2}{\sigma_m}$$ **[5 marks]**

Where: *m*- market portfolio
w- weight of asset in the portfolio
σ^2 - variance
σ_{im} - covariance between the returns of the market portfolio and returns from security *i*

Hence or otherwise prove that, at equilibrium, if a risk free (R_f) security exists

$$E(\tilde{R}_i) = R_f + [E(\tilde{R}_m) - R_f]\frac{\sigma_{im}}{\sigma_m^2}$$ **[5 marks]**

iii. Suppose you want to conduct a cross-sectional test of the CAPM, using Fama-MacBeth equation as stated below:

$$\overline{R}_i = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 CHAR_i + \delta_i$$

If the CAPM is true what would be your expectation of each of the above coefficients, i.e. γ's. **[6 marks]**

iv. If the regression data in (*iv)* above is inconsistent with the CAPM, there are four possible situations that you may expect to find. State and explain any **two (2)** of these situations (use diagrammatical illustrations to explain your answer). **[4 marks]**

Question Four

i. Find the weights of the two pure factor portfolios constructed from the following three securities:

$$r_1 = 0.06 + 2F_1 + 2F_2$$
$$r_2 = 0.05 + 3F_1 + 1F_2$$
$$r_3 = 0.04 + 3F_1 + 0F_2$$

Assuming a risk-free rate of 5% and no arbitrage, write the factor equations for the two pure factor portfolios, and determine their risk premiums. **[9; 6 marks]**

ii. Assuming the factor model in (*i*) above applies, if there exist an additional asset with the following factor equation,

$$r_4 = 0.08 + 1F_1 + 0F_2$$

Determine whether an arbitrage opportunity exists? If it exists, describe how you would take advantage of it. **[6, 4 marks]**

Question Five

i. Explain the principle of risk neutral valuation. **[4 marks]**

ii. State the mathematical definition of the Brownian motion. **[5 marks]**

iii. Let *p* be the price of a European put option on a stock which pays no dividends; let *X* be the strike price of the option, *S* be the spot price of the stock, *T* be the time to the option's expiration (in years) and *r* be the *T*–year spot interest rate. By comparing two portfolios consisting of options, cash and stock, prove that

$$P > Xe^{-rT} - S$$ **[10 marks]**

iv. Critically examine any **three (3)** limitations of the Black-Scholes option valuation model in pricing of derivatives from a developing economy perspective. **[6 marks]**

END OF EXAMINATION PAPER