NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

CFI 5211

FACULTY OF COMMERCE DEPARTMENT OF FINANCE

MSc FINANCE AND INVESTMENTS

CFI 5211: FINANCIAL ENGINEERING

NOV/DEC 2015 EXAMINATION

Time : 3 hours

INSTRUCTIONS:

Candidates should attempt **ALL QUESTIONS.**

This paper carries 100 Marks.

Statistical Tables are attached at the end of the question paper.

QUESTION 1 **[20]**

The forward price of an investment asset providing no income is given by

$$F_0 = S_0 e^{rT},$$

where T is time to maturity, r is the risk-free rate of interest and S_0 is the asset price at time $t = 0$. Consider a four-month forward contract to buy an ounce of gold currently valued at \$1300. Assume that the four-month risk-free rate of interest is 6% per annum.

(a) Obtain the forward price. [3]

(b) What would arbitrageurs do if $F_0 \neq S_0 e^{rT}$? [3]

(c) Suppose f is the value of a long forward contract that has a delivery price of K. The value f is generally given by

$$f = (F_0 - K)e^{-rT}.$$

If the delivery price is \$1320, calculate the value of the long forward contract. [4]

Look at the spot interest rates shown in the following Table:

Year	Spot rate
1	$Y(t; t+1) \equiv r_1 = 0.050$
2	$Y(t; t+2) \equiv r_2 = 0.045$
3	$Y(t; t+3) \equiv r_3 = 0.040$
4	$Y(t; t+4) \equiv r_4 = 0.035$
5	$Y(t; t+5) \equiv r_5 = 0.030$

Suppose that someone told you that the 6-year spot interest rate was 5.50 percent.

(a) Would you believe him or not? Why? [4]

(b) Could you make money if he was right? How? [2]

(c) What is the sensible value for the 6-year spot rate? [4]

QUESTION 2 **[20]**

Consider a simple discrete-time model with $T = 2$ and four states of the world. Suppose $r = 4\%$ and the risky security is as follows:

ω_k	$t = 0$	$t = 1$	$t = 2$
ω_1	$S_0 = 10$	$S_1 = 12$	$S_2 = 13$
ω_2	$S_0 = 10$	$S_1 = 12$	$S_2 = 10$
ω_3	$S_0 = 10$	$S_1 = 8$	$S_2 = 10$
ω_4	$S_0 = 10$	$S_1 = 8$	$S_2 = 7$

(a) Draw the resulting tree diagram. [2]

(b) Find Q_u and Q_d. [3]

(c) Find Q_{uu} and Q_{ud}. [3]

(d) Find Q_{du} and Q_{dd}. [3]

(e) Hence, by showing all the necessary steps, find the discrete time martingale measure Q. [4]

(f) What can you conclude regarding the resulting trading strategy? [2]

QUESTION 3 **[20]**

The simple forward rate or LIBOR forward rate L for $[S, T]$ contracted at time t, is the solution to the equation

$$1 \cdot (1 + (T - S) \cdot L) = 1 \cdot \frac{p(t, S)}{p(t, T)}$$

where time T is the maturity time of the forward LIBOR, $T - S$ is called the tenor and $1/(T - S)$ is the "accrued factor" or the "day-count fraction".

(a) Deduce an equation for $L(t, S, T)$. [6]

(b) Hence, deduce an equation for L when $t = S$ in (a). [4]

(c) What is the name given to the process in (b)? [1]

Consider a European call option on a non-dividend paying stock where the stock price is \$51, the exercise price is \$50, the time to maturity is 16 months and the risk-free rate is 1% per month.

(d) Find an upper bound for the option price. [5]

(f) Find a lower bound for the option price. [4]

QUESTION 4 **[20]**

Consider an American option with the following specifications:

• The underlying asset is currently valued at \$120.

• The strike price is set at \$120.

• The risk-free rate is 5% per annum and the riskiness of investing in the underlying is given as 30%. Consider a time partition of 1 year and a maturity of 3 years.

(a) Calculate the down and up movement factors. [2]

(b) Calculate the down and up probabilities. [2]

(c) Hence, calculate all the underlying asset prices. [6]

(d) What is the fair price you should pay to have a **call** option document drafted? Show all the working. [10]

QUESTION 5 **[20]**

The Black-76 formula for a caplet is given by

$$Capl_i^B(t) = \alpha_i p_i(t)\{L_i(t)N[d_1] - RN[d_2]\}$$

where

$$\begin{aligned} d_1 &= \frac{1}{\sigma_i\sqrt{T_i - t}}\left\{\ln\left(\frac{L_i(t)}{R}\right) + \frac{\sigma_i^2}{2}\cdot(T_i - t)\right\} \\ d_2 &= \frac{1}{\sigma_i\sqrt{T_i - t}}\left\{\ln\left(\frac{L_i(t)}{R}\right) - \frac{\sigma_i^2}{2}\cdot(T_i - t)\right\} \\ &= d_1 - \sigma_i\sqrt{T_i - t} \end{aligned}$$

Consider a caplet on LIBOR with the following specifications:

o The notional amount is $N = \$10\ 000\ 000$.

o $T_i - t = 0.25$ years

o Current quoted (clean) LIBOR is 6%, constituting two-thirds of the (clean) caplet rate.

o The 3 month risk free interest rate is 3% per annum.

o The volatility of the forward LIBOR is estimated at 6% per annum.

(a) Knowing that the bond price is given by $p(t) = N\exp(-rT)$, compute $p(t)$. [2]

(b) Calculate the accrual factor. [2]

(c) Find the necessary option parameters. [4]

(d) Hence, by showing **all** the necessary steps, find the premium on the caplet. [10]

(e) Under what circumstances would you exercise the caplet at maturity? [2]

END OF EXAMINATION PAPER