NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF INDUSTRIAL TECHNOLOGY

BACHELOR OF ENGINEERING (HONS) DEGREE

Final examinations
January 2011

TEE 2101 **Network Theory**

Duration of Examination 3 Hours

Instructions to candidates:
1. Answer any <u>five</u> questions only.
2. Each question carries equal marks.
3. Draw the circuits clearly and explain all your steps in any solution.
4. Start the answers for the new question on a fresh page.

<u>QUESTION 1</u>

Find $v_o(t)$ using the superposition theorem in the circuit of Figure Q1. [20]

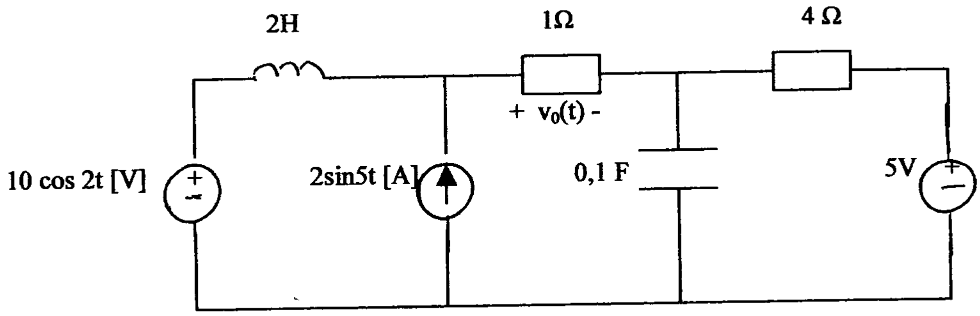

Figure Q1

<u>QUESTION 2</u>

The switch in the circuit below in figure Q2 has been closed for a long time. At t = 0 the switch is opened. Find i(t) for t >0. [20]

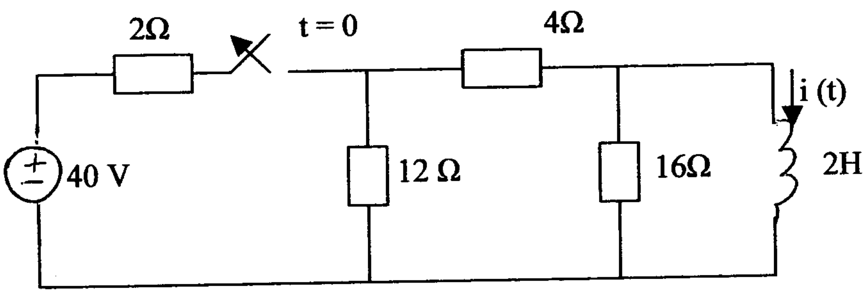

Figure Q2

QUESTION 3

Find i(t) in the circuit of Figure Q3 for $t > 0$. Assume that the switch has been closed for a long time. [20]

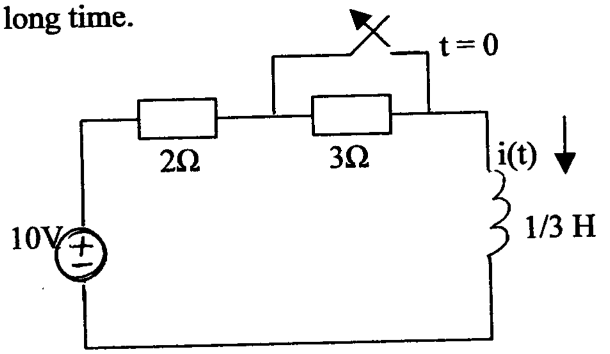

Figure Q3

QUESTION 4

a) Find the sinusoids represented by phasors $V = j8\,e^{-j20}$ and $I = -3 + j4$ [6]

b) Transform sinusoids $v = -4\sin(30t + 50^\circ)$ and $i = 6\cos(50t - 40^\circ)$ to phasors. [4]

c) Using the phasor approach, determine the current i(t) in a circuit described by the integro-differential equation:

$$4i + 8\int i\,dt - 3\frac{di}{dt} = 50\cos(2t + 75^\circ)$$ [10]

QUESTION 5

In the parallel circuit in figure Q5 calculate resonance frequency ω_o, quality factor Q, bandwidth B, half frequencies ω_1, ω_2 and determine the power dissipated at $\omega_o, \omega_1, \omega_2$. [20]

10 sin wt

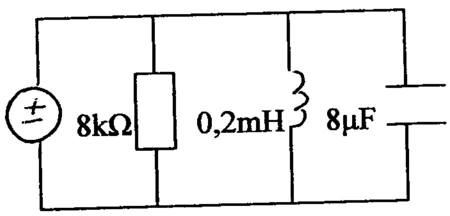

Figure Q5

QUESTION 6

a) Find [z] and [g] parameters of a two-port network if: [8]

$$T = \begin{vmatrix} 10 & 1{,}5\Omega \\ 2s & 4 \end{vmatrix}$$

b) Obtain the inverse Fourier Transform of the function: $F(w) = \dfrac{10jw+4}{(jw)^2+6jw+8}$ [12]

QUESTION 7

a) Find the exponential Fourier series expansion of the periodic function: [10]

$f(t) = e^t, \quad 0 < t < 2\pi$ with $f(t+2\pi) = f(t)$

b) Find $f(t)$ given that $F(s) = \dfrac{s^2+12}{s(s+2)(s+3)}$ [10]

QUESTION 8

For the circuit in Figure Q8 find v(t) and i(t) for t > 0. [20]

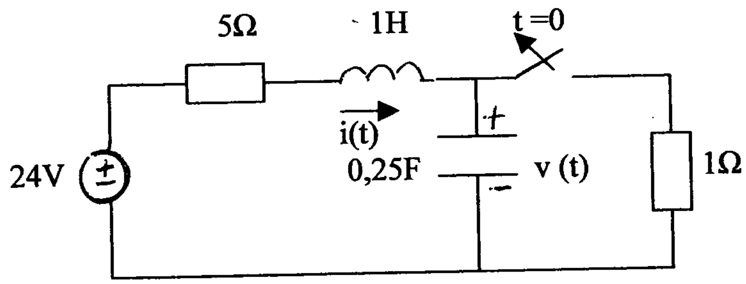

Figure Q8

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TABLE 15.1 Properties of the Laplace transform.

Property	$f(t)$	$F(s)$
Linearity	$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(s) + a_2 F_2(s)$
Scaling	$f(at)$	$\frac{1}{a} F\left(\frac{s}{a}\right)$
Time shift	$f(t-a)u(t-a)$	$e^{-as} F(s)$
Frequency shift	$e^{-at} f(t)$	$F(s+a)$
Time differentiation	$\frac{df}{dt}$	$sF(s) - f(0^-)$
	$\frac{d^2 f}{dt^2}$	$s^2 F(s) - sf(0^-) - f'(0^-)$
	$\frac{d^3 f}{dt^3}$	$s^3 F(s) - s^2 f(0^-) - sf'(0^-) - f''(0^-)$
	$\frac{d^n f}{dt^n}$	$s^n F(s) - s^{n-1} f(0^-) - s^{n-2} f'(0^-) - \cdots - f^{(n-1)}(0^-)$
Time integration	$\int_0^t f(t)\,dt$	$\frac{1}{s} F(s)$
Frequency differentiation	$tf(t)$	$-\frac{d}{ds} F(s)$
Frequency integration	$\frac{f(t)}{t}$	$\int_s^\infty F(s)\,ds$
Time periodicity	$f(t) = f(t + nT)$	$\frac{F_1(s)}{1 - e^{-sT}}$
Initial value	$f(0^+)$	$\lim_{s \to \infty} sF(s)$
Final value	$f(\infty)$	$\lim_{s \to 0} sF(s)$
Convolution	$f_1(t) * f_2(t)$	$F_1(s) F_2(s)$

TABLE 15.2 Laplace transform pairs.

$f(t)$	$F(s)$
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
e^{-at}	$\frac{1}{s+a}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
te^{-at}	$\frac{1}{(s+a)^2}$
$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
$\sin(\omega t + \theta)$	$\frac{s \sin \theta + \omega \cos \theta}{s^2 + \omega^2}$
$\cos(\omega t + \theta)$	$\frac{s \cos \theta - \omega \sin \theta}{s^2 + \omega^2}$
$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2 + \omega^2}$
$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2 + \omega^2}$

TABLE 17.1 Properties of the Fourier transform.

Property	$f(t)$	$F(\omega)$
Linearity	$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(\omega) + a_2 F_2(\omega)$
Scaling	$f(at)$	$\frac{1}{\lvert a\rvert} F\left(\frac{\omega}{a}\right)$
Time shift	$f(t-a)u(t-a)$	$e^{-j\omega a} F(\omega)$
Frequency shift	$e^{j\omega_0 t} f(t)$	$F(\omega - \omega_0)$
Modulation	$\cos(\omega_0 t)\, f(t)$	$\frac{1}{2}[F(\omega+\omega_0) + F(\omega - \omega_0)]$
Time differentiation	$\frac{df}{dt}$	$j\omega F(\omega)$
	$\frac{d^n f}{dt^n}$	$(j\omega)^n F(\omega)$
Time integration	$\int_{-\infty}^{t} f(t)\, dt$	$\frac{F(\omega)}{j\omega} + \pi F(0)\,\delta(\omega)$
Frequency differentiation	$t^n f(t)$	$(j)^n \frac{d^n}{d\omega^n} F(\omega)$
Reversal	$f(-t)$	$F(-\omega)$ or $F^*(\omega)$
Duality	$F(t)$	$2\pi f(-\omega)$
Convolution in t	$f_1(t) * f_1(t)$	$F_1(\omega) F_2(\omega)$
Convolution in ω	$f_1(t) f_1(t)$	$\frac{1}{2\pi} F_1(\omega) * F_2(\omega)$

TABLE 17.2 Fourier transform pairs.

$f(t)$	$F(\omega)$
$\delta(t)$	1
1	$2\pi\delta(\omega)$
$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$
$u(t+\tau) - u(t-\tau)$	$2\frac{\sin\omega\tau}{\omega}$
$\lvert t\rvert$	$\frac{-2}{\omega^2}$
$\text{sgn}(t)$	$\frac{2}{j\omega}$
$e^{-at}u(t)$	$\frac{1}{a + j\omega}$
$e^{at}u(-t)$	$\frac{1}{a - j\omega}$
$t^n e^{-at} u(t)$	$\frac{n!}{(a+j\omega)^{n+1}}$
$e^{-a\lvert t\rvert}$	$\frac{2a}{a^2+\omega^2}$
$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$
$\sin\omega_0 t$	$j\pi[\delta(\omega+\omega_0) - \delta(\omega-\omega_0)]$
$\cos\omega_0 t$	$\pi[\delta(\omega+\omega_0) + \delta(\omega-\omega_0)]$
$e^{-at}\sin\omega_0 t\, u(t)$	$\frac{\omega_0}{(a+j\omega)^2+\omega_0^2}$
$e^{-at}\cos\omega_0 t\, u(t)$	$\frac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$

TABLE 18.1 Conversion of two-port parameters.

	z		y		h		g		T		t	
z	z_{11}	z_{12}	$\frac{y_{22}}{\Delta_y}$	$-\frac{y_{12}}{\Delta_y}$	$\frac{\Delta_h}{h_{22}}$	$\frac{h_{12}}{h_{22}}$	$\frac{1}{g_{11}}$	$-\frac{g_{12}}{g_{11}}$	$\frac{A}{C}$	$\frac{\Delta_T}{C}$	$\frac{d}{c}$	$\frac{1}{c}$
	z_{21}	z_{22}	$-\frac{y_{21}}{\Delta_y}$	$\frac{y_{11}}{\Delta_y}$	$-\frac{h_{21}}{h_{22}}$	$\frac{1}{h_{22}}$	$\frac{g_{21}}{g_{11}}$	$\frac{\Delta_g}{g_{11}}$	$\frac{1}{C}$	$\frac{D}{C}$	$\frac{\Delta_t}{c}$	$\frac{a}{c}$
y	$\frac{z_{22}}{\Delta_z}$	$-\frac{z_{12}}{\Delta_z}$	y_{11}	y_{12}	$\frac{1}{h_{11}}$	$-\frac{h_{12}}{h_{11}}$	$\frac{\Delta_g}{g_{22}}$	$\frac{g_{12}}{g_{22}}$	$\frac{D}{B}$	$-\frac{\Delta_T}{B}$	$\frac{a}{b}$	$-\frac{1}{b}$
	$-\frac{z_{21}}{\Delta_z}$	$\frac{z_{11}}{\Delta_z}$	y_{21}	y_{22}	$\frac{h_{21}}{h_{11}}$	$\frac{\Delta_h}{h_{11}}$	$-\frac{g_{21}}{g_{22}}$	$\frac{1}{g_{22}}$	$-\frac{1}{B}$	$\frac{A}{B}$	$-\frac{\Delta_t}{b}$	$\frac{d}{b}$
h	$\frac{\Delta_z}{z_{22}}$	$\frac{z_{12}}{z_{22}}$	$\frac{1}{y_{11}}$	$-\frac{y_{12}}{y_{11}}$	h_{11}	h_{12}	$\frac{g_{22}}{\Delta_g}$	$-\frac{g_{12}}{\Delta_g}$	$\frac{B}{D}$	$\frac{\Delta_T}{D}$	$\frac{b}{a}$	$\frac{1}{a}$
	$-\frac{z_{21}}{z_{22}}$	$\frac{1}{z_{22}}$	$\frac{y_{21}}{y_{11}}$	$\frac{\Delta_y}{y_{11}}$	h_{21}	h_{22}	$-\frac{g_{21}}{\Delta_g}$	$\frac{g_{11}}{\Delta_g}$	$-\frac{1}{D}$	$\frac{C}{D}$	$\frac{\Delta_t}{a}$	$\frac{c}{a}$
g	$\frac{1}{z_{11}}$	$-\frac{z_{12}}{z_{11}}$	$\frac{\Delta_y}{y_{22}}$	$\frac{y_{12}}{y_{22}}$	$\frac{h_{22}}{\Delta_h}$	$-\frac{h_{12}}{\Delta_h}$	g_{11}	g_{12}	$\frac{C}{A}$	$-\frac{\Delta_T}{A}$	$\frac{c}{d}$	$-\frac{1}{d}$
	$\frac{z_{21}}{z_{11}}$	$\frac{\Delta_z}{z_{11}}$	$-\frac{y_{21}}{y_{22}}$	$\frac{1}{y_{22}}$	$-\frac{h_{21}}{\Delta_h}$	$-\frac{h_{11}}{\Delta_h}$	g_{21}	g_{22}	$\frac{1}{A}$	$\frac{B}{A}$	$\frac{\Delta_t}{d}$	$-\frac{b}{d}$
T	$\frac{z_{11}}{z_{21}}$	$\frac{\Delta_z}{z_{21}}$	$-\frac{y_{22}}{y_{21}}$	$-\frac{1}{y_{21}}$	$-\frac{\Delta_h}{h_{21}}$	$-\frac{h_{11}}{h_{21}}$	$\frac{1}{g_{21}}$	$\frac{g_{22}}{g_{21}}$	A	B	$\frac{d}{\Delta_t}$	$\frac{b}{\Delta_t}$
	$\frac{1}{z_{21}}$	$\frac{z_{22}}{z_{21}}$	$-\frac{\Delta_y}{y_{21}}$	$-\frac{y_{11}}{y_{21}}$	$-\frac{h_{22}}{h_{21}}$	$-\frac{1}{h_{21}}$	$\frac{g_{11}}{g_{21}}$	$\frac{\Delta_g}{g_{21}}$	C	D	$\frac{c}{\Delta_t}$	$\frac{a}{\Delta_t}$
t	$\frac{z_{22}}{z_{12}}$	$\frac{\Delta_z}{z_{12}}$	$-\frac{y_{11}}{y_{12}}$	$-\frac{1}{y_{12}}$	$\frac{1}{h_{12}}$	$\frac{h_{11}}{h_{12}}$	$-\frac{\Delta_g}{g_{12}}$	$-\frac{g_{22}}{g_{12}}$	$\frac{D}{\Delta_T}$	$\frac{B}{\Delta_T}$	a	b
	$\frac{1}{z_{12}}$	$\frac{z_{11}}{z_{12}}$	$-\frac{\Delta_y}{y_{12}}$	$-\frac{y_{22}}{y_{12}}$	$\frac{h_{22}}{h_{12}}$	$\frac{\Delta_h}{h_{12}}$	$-\frac{g_{11}}{g_{12}}$	$-\frac{1}{g_{12}}$	$\frac{C}{\Delta_T}$	$\frac{A}{\Delta_T}$	c	d

$\Delta_z = z_{11}z_{22} - z_{12}z_{21}$, $\Delta_h = h_{11}h_{22} - h_{12}h_{21}$, $\Delta_T = AD - BC$

$\Delta_y = y_{11}y_{22} - y_{12}y_{21}$, $\Delta_g = g_{11}g_{22} - g_{12}g_{21}$, $\Delta_t = ad - bc$