NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA1102

FACULTY OF APPLIED SCIENCE

DEPARTMENT OF APPLIED MATHEMATICS

SMA1102 LINEAR ALGEBRA

AUGUST 2024 SUPP EXAMINATION

Time : 3 hours

This paper contains **TWO** sections. Attempt **ALL** questions from Section A and any **THREE** questions from Section B

SECTION A: Answer ALL questions in this section [40].

Attempt **ALL** questions from this section [**40 MARKS**]

A1. Define the following;

(a) Singular matrix, [2]

(b) Diagonal matrix, [2]

(c) Equal vectors. [2]

A2. Find the equation of the plane passing through the points $P_1(1, 2, -1)$, $P_2(2, 3, 1)$, and $P_3(3, -1, 2)$. [4]

A3. Prove that if B and C are both inverses of the matrix A, then $B = C$. [4]

A4. Verify that if $A\mathbf{X} = \mathbf{0}$ is a homogeneous linear system of m equations in n unknowns then the set of solution vectors is a subspace of $\mathbb{R}^n$. [5]

A5. Use Gauss-Jordan elimination to solve the following system of linear equations

$$\begin{aligned} -2x_3 + 7x_5 &= 12, \\ 2x_1 + 4x_2 - 10x_3 + 6x_4 + 12x_5 &= 28, \\ 2x_1 + 4x_2 - 5x_3 + 6x_4 - 5x_5 &= -1. \end{aligned}$$

[6]

A6. Let $U = \{(x, y, z) \in \mathbb{R}^3 | x, y, z \in \mathbb{R}\}, V = \{(x, y, z) \in \mathbb{R}^3 | x + 2yz = 6\}$ and $W = \{(a + b, a^2, a - b) \in \mathbb{R}^3 | a, b \in \mathbb{R}\}$. For each of the following vectors, decide which (if any) of U, V or W the vector is in;

(a) $(-2, 1, 3)$, [4]

(b) $(\frac{2}{3}, \frac{1}{9}, 0)$. [4]

A7. Determine which of the following lists of vectors are linearly independent or not.

(a) $\{(1,\ 3)\}$, [2]

(b) $\{(1,\ 0,\ 0,\ 0,\ 0),\ (1,\ 1,\ 0,\ 0,\ 0),\ (1,\ 1,\ 1,\ 0,\ 0),\ (1,\ 1,\ 1,\ 1,\ 0),\ (1,\ 1,\ 1,\ 1,\ 1)\}$, [2]

(c) $\{(4,\ 8,\ 12),\ (1.5,\ 1,\ 0.5),\ (4,\ 2,\ 6)\}$, [3]

SECTION B: Answer THREE questions in this section [60].

Answer any **THREE** questions from this section **[60 MARKS]**

B8. (a) Find the area of the triangle having vertices $A(1, 0, 1),\ B(0, 2, 3),$ and $C(2, 1, 0)$. [5]

(b) (i) Show that

$$\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}.$$

[3]

(ii) Hence calculate the scalar triple product $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$ of the vectors $\mathbf{v} = 5\mathbf{i} + 2\mathbf{j} - 7\mathbf{k},\ \mathbf{w} = 3\mathbf{i} - 9\mathbf{j},$ and $\mathbf{u} = \mathbf{i} + 4\mathbf{j} + 8\mathbf{k}$. [2]

(c) Given two planes $x + 2y - 2z = 3$ and $2x + 4y - 4z = 7$,

(i) show that the two planes are parallel. [2]

(ii) Find the distance between the two planes. [3]

(d) Find the acute angle between the plane $x - y - 3z = 5$ and the line $x = 2 - t$, $y = 2t,\ z = 3t - 1$ to the nearest degree. [5]

B9. (a) Use elementary matrices to compute the inverse of matrix

$$H = \begin{pmatrix} 1 & 1 & -2 \\ 0 & 1 & 1 \\ 0 & -1 & 3 \end{pmatrix}.$$

[10]

(b) Consider the following system of equations,

$$\begin{aligned} w + x + 2y &= a, \\ x + y + 2z &= 0, \\ w + x + 3y + 3z &= 0, \\ 2x + 5y + bz &= 3. \end{aligned}$$

(i) find the values of a and b for which the system has a unique solution, [5]

(ii) and find the point(s) (a, b) such that the system has at least two solutions. [5]

B10. (a) Define a vector space. [8]

(b) Let $V = R^2$ and define addition and scalar multiplication operations as follows; If $\mathbf{u} = (u_1, u_2)$ and $\mathbf{v} = (v_1, v_2)$, then define $\mathbf{u} + \mathbf{v} = (u_1 + v_1, u_2 + v_2)$ and if k is any scalar, then define $k\mathbf{u} = (ku_1, 0)$. Show that V is not a vector space with the stated operations. [4]

(c) Prove that if W is a set of one or more vectors from a vector space V, then W is a subspace of V if and only if W is closed under addition and closed under scalar multiplication. [5]

(d) Show that a line through the origin of $\mathbb{R}^3$ is a subspace of $\mathbb{R}^3$. [3]

B11. (a) Express $\mathbf{v} = (3, 7, -4)$ in $\mathbb{R}^3$ as a linear combination of the vectors $\mathbf{v}_1 = (1, 2, 3)$, $\mathbf{v}_2 = (2, 3, 7)$, $\mathbf{v}_3 = (3, 5, 6)$. [4]

(b) Prove that if $S = \{\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_n\}$ is a basis for a vector space V, then every vector $\mathbf{v}$ in V can be expressed in the form $\mathbf{v} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \cdots + c_n\mathbf{v}_n$ in exactly one way. [5]

(c) Find the basis and dimension of the subspace $V = \{\mathbf{x} \in \mathbb{R}^4 | A\mathbf{x} = \mathbf{0}\}$, where

$$A = \begin{pmatrix} 1 & 1 & -2 & 4 \\ 2 & 2 & -3 & 1 \\ 3 & 3 & -4 & -2 \end{pmatrix}.$$

[5]

(d) Let U be the subspace of $\mathbb{R}^4$ spanned by vectors $\mathbf{u}_1 = (1, -2, 5, -3)$, $\mathbf{u}_2 = (2, 3, 1, -4)$, $\mathbf{u}_3 = (3, 8, -3, -5)$.

(i) Find a basis for U, and [4]

(ii) extend the basis to a basis for $\mathbb{R}^4$. [2]

END OF QUESTION PAPER