NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA1103

FACULTY OF APPLIED SCIENCE

DEPARTMENT OF APPLIED MATHEMATICS

SMA1103: DISCRETE MATHEMATICS

December 2024: Examination

Time : 3 hours

Candidates should attempt **ALL** questions from **Section A** (40 marks) and **ANY THREE** questions from **Section B** (20 marks each).

SECTION A [40]

A1.   (a) Define the following:

(i) two equal sets A and B. [2]

(ii) a symmetric relation. [2]

(b) Find the inverse of 7 mod 9. [4]

A2.   (a) Prove that $(-x)(-y) = xy$. [5]

(b) Show that $(\neg(p \wedge q) \wedge q) \equiv (\neg p \wedge q)$. [5]

A3.   (a) Find the exact fraction for $7.3\overline{169}$. [4]

(b) Prove that if $K \equiv 2(mod3)$, then $K^3 \equiv 8(mod9)$. [5]

A4. (a) Find the closed form for the sequence $u_0 = 2, u_1 = 2$ and
$u_{n+2} = 6u_{n+1} - 9u_n$. [5]

(b) Let $A = \{-1, 0, 1\}$ and $B = \{0, 2, 6, 6\}$. Compute the following

(i) Power set of A. [2]

(ii) $|A \times B|$. [2]

(iii) $A - B$. [1]

(iv) $B - A$. [1]

(v) $A \triangle B$. [2]

SECTION B [60]

B5. (a) (i) Show that $f = \{(x, 3x + 7) : x \in \Re\}$ is a one-to-one function. [3]

(ii) Find f^{-1} for f given above. [2]

(iii) Using $f(x) = 6 - 3x$ and $g(x) = \dfrac{3}{7 - 2x}$, verify
that $(f o g)^{-1}(x) = (g^{-1} o f^{-1})(x)$. [6]

(b) Prove that the congruence modulo n, $x \equiv y (mod\, n)$, if $n/(x - y)$, is
an equivalence relation. [6]

(c) Find the exact value of $\dfrac{n!}{(n - 5)!} = 20\dfrac{n!}{(n - 3)!}$. [3]

B6. (a) Given that set $S = \{1, 3, 6, 9\}$ and $V = \{2, 3, 6, 9\}$,

(i) define the symmetric difference of two sets in words. [2]

(ii) write down the partition of set S. [2]

(b) Draw a Venn diagram and write using $\cup$ and/or $\cap$ in the following cases

(i) $A \backslash B$. [4]

(ii) $A \oplus B$. [4]

(c) Solve $140x \equiv 56 (mod\, 252)$. [8]

B7. (a) (i) Prove by induction, that $\sum_{k=1}^{n}(2k-1) = n^2$. [4]

(ii) Show that $A - (B \cup C) = (A - B) \cap (A - C)$. [3]

(iii) Prove that the conditional operation distributes over the operation of conjunction: $p \to (q \wedge r) \equiv (p \to q) \wedge (p \to r)$. [5]

(b) (i) Use Euclid's algorithm to determine the greatest common divisor (gcd) of 143 and 559. [4]

(ii) Show that the greatest common divisor can be written as $gcd(143, 559) = \alpha(143) + \beta(559)$, where α and β are real constants. [4]

B8. (a) State without the proof the completeness axiom. [2]

(b) Let A be a set and let $\{B_i\}_{i \in \mathbb{N}}$ be a family of sets. Show that
$A - \cap_{i \in \mathbb{N}} B_i = \cup_{i \in \mathbb{N}} (A - B_i)$. [6]

(c) ZIMSTATS wishes to conduct a survey on income levels for two randomly selected cities and two new resettlement areas. Eleven employees are to conduct the survey. Two employees will be deployed to each city, three to one of the resettlements and four to the other resettlement due to problems of accessibility. Find the number of ways the employees may be deployed. [6]

(d) Prove $\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$. [6]

END OF QUESTION PAPER