

NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY
SMA 1111

FACULTY OF APPLIED SCIENCES
DEPARTMENT OF APPLIED MATHEMATICS
SMA1111: MATHEMATICS FOR SCIENCE 1

DECEMBER 2024 EXAMINATION

Time : 3 hours

Candidates should attempt **ALL** questions from Section A[40 MARKS] and **ANY THREE** questions from Section B[60 MARKS].

SECTION A [40 marks]

- A1.** (a) State the first principle definition of a derivative. [2]
(b) Use the definition of a derivative to show that $f(x) = \sin x$ is differentiable everywhere and that $f'(x) = \cos x$. [4]

A2. Evaluate the following limits

(a) $\lim_{x \rightarrow \infty} \frac{1+x}{2+x}$. [2]

(b) $\lim_{x \rightarrow \infty} \left(1 - \frac{3}{x}\right)^{2x}$. [5]

A3. Given that

$$A = \begin{pmatrix} 3 & 2 & 1 \\ 2 & 3 & 1 \\ 2 & 1 & 3 \end{pmatrix},$$

- (a) determine whether matrix A is invertible, [2]
(b) if matrix A is invertible then find its inverse. [4]

A4. Evaluate the following

(a) $\int x^2 e^{x^3+6} dx.$ [3]

(b) $\int_0^6 f(x) dx$ given that $f(x) = \begin{cases} x^2 & \text{if } x < 2, \\ 3x - 2 & \text{if } x \geq 2. \end{cases}$ [3]

(c) $\int \frac{2x}{2x^2 + x - 1} dx.$ [4]

A5. (a) Define continuity of a function $f(t)$ at point $t = a.$ [3]

(b) Determine if the function $f(t) = \frac{1+t^3}{1-t^2}$ is continuous at $t = -1.$ [4]

A6. Determine and classify the critical points for the function $f(x) = x^3 - 3x^2 - 9x + 2$ in the closed interval $[2, 5].$ [4]

SECTION B

ANSWER ANY THREE QUESTIONS [60 MARKS]

B7. (a) Expand the function $f(x) = xe^{4x}$ about $x = 0$ using the Maclaurin series and obtain the first three non-zero terms. [7]

(b) Evaluate $\sqrt{5}$ using the Newton's method, correct to four decimal places. [5]

(c) Determine the area of the region bounded by $y = 4x - x^2$ and the x -axis. [4]

(d) Let

$$f(x) = \begin{cases} \frac{a}{x^2} & \text{if } x \geq 3, \\ \frac{x^3}{3} & \text{if } x < 3. \end{cases}$$

Determine the value of a such that the function $f(x)$ is continuous at $x = 3.$ [4]

B8. (a) Given the system

$$\begin{aligned} x + 2y - 3z &= 4 \\ 3x - y + 5z &= 2 \\ 4x + y + (a^2 - 14)z &= a + 2 \end{aligned}$$

Reduce the matrix to row echelon form and find the values of a for which the system has

- (i) no solution,
 (ii) unique solution,
 (iii) infinitely many solutions.

[2,2,2,2]

(b) (i) State DeMoivre's theorem. [1]

(ii) Use DeMoivre's theorem to evaluate $\int_0^2 e^{3xi} dx$. [3]

(iii) Express $\cos 5\theta$ in terms of $\cos \theta$ and hence solve the equation $16x^5 - 20x^3 + 5x - 1 = 0$. [8]

B9. (a) Solve the following system of equations using

(i) The Gaussian elimination method. [6]

(ii) The Cramers rule. [6]

$$\begin{aligned} -2x_1 + x_2 - x_3 &= 4 \\ x_1 + 2x_2 + 3x_3 &= 13 \\ 3x_1 + x_3 &= -1 \end{aligned}$$

(b) Verify your answer in (a) by showing that $\mathbf{X} = \mathbf{A}^{-1}\mathbf{B}$. [4]

(c) Use the trapezoidal rule with $n = 4$ to approximate $\int_{-1}^3 \sin x^2 dx$. [4]

B10. (a) Evaluate the following integrals

(i) $\int_{-1}^3 xe^{2x} dx$. [6]

(ii) $\int \frac{x}{\sqrt{4-x^2}} dx$. [4]

(b) Differentiate $y = \sin^{-1} x$ with respect to x . [3]

(c) Find T^{-1} using the inverse method, given that $T = \begin{pmatrix} 3 & 2 & -1 \\ 1 & 6 & 3 \\ 2 & -4 & 0 \end{pmatrix}$. [7]

END OF QUESTION PAPER