NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA1112

FACULTY OF APPLIED SCIENCE

DEPARTMENT OF APPLIED MATHEMATICS

SMA1112: PREPARATORY MATHEMATICS

DECEMBER 2024 EXAMINATION

Time : 3 hours

Candidates should attempt **ALL** questions from Section A[**40 MARKS**] and **ANY THREE** questions from Section B[**60 MARKS**].

SECTION A (40 MARKS)

A1. (a) Evaluate $\left(\frac{1}{4}\right)^{-\frac{1}{2}}$. [2]

(b) Simplify

$$\frac{\left(\sqrt{xy}\right)^3 \times x^{\frac{1}{2}}}{x\sqrt{y}},$$

where x and y are positive real numbers. [3]

(c) Solve the equation,

$$4^x + 2^{x+1} - 8 = 0.$$

[4]

A2. (a) Given that $f(x) = 2x + 1$ and $g(x) = x + 3$. Find,

(i) the value of x for which $f(x) = g(x)$, [1]

(ii) $f\left(g(x)\right)$. [2]

(b) A quadratic polynomial is defined by $p(x) = ax^2 + bx + c$, write down a condition in terms of a, b and c for which $p(x)$ is a perfect square. [2]

(c) Express $4x^2 - 12x - 7$ in the form $(px + q)^2 + r$, hence or otherwise solve the equation,

$$4x^2 - 12x - 7 = 0.$$

[4]

A3. (a) The hypotenuse of a right-angled isosceles triangle is x units, determine the area of the triangle in terms of x. [3]

(b) Find the exact values of $\sin\theta$ and $\tan\theta$ given that $\sec\theta = 2$. [3]

A4. (a) Given that $f(x) = x^3 - 7x + 6$, show that $(x-2)$ is a factor of $f(x)$ and hence factorise $f(x)$ completely. [4]

(b) Decompose the fraction $\dfrac{7x+1}{x^3-7x+6}$ into partial fractions. [5]

(c) It is given that $(1+x)^n = \displaystyle\sum_{r=0}^{n} \binom{n}{r} x^r$ where $n \in \mathbb{Z}^+$.

Derive a formula for the expansion of $(a+b)^n$ and hence determine the term independent of x in the expansion of,

$$\left(x + \frac{1}{x^2}\right)^9.$$

[7]

SECTION B

B5. (a) Find the inverse of the following matrix,

$$\mathbf{A} = \begin{pmatrix} 3 & -1 & 2 \\ 1 & 2 & 4 \\ 2 & -1 & 1 \end{pmatrix}.$$

[7]

(b) Use the Cramer's rule to solve the following system of equations

$$\begin{aligned} 3x - y + 2z &= 7, \\ x + 2y + 4z &= 17, \\ 2x - y + z &= 3. \end{aligned}$$

[6]

(c) Verify your answer in part (b) by showing that $\mathbf{X} = \mathbf{A}^{-1}\mathbf{B}$, where $\mathbf{A}^{-1}$ is the inverse of matrix $\mathbf{A}$ in (a) and,

$$\mathbf{B} = \begin{pmatrix} 7 \\ 17 \\ 3 \end{pmatrix}.$$

[4]

(d) Given a system of linear equations in matrix form as $\mathbf{AX} = \mathbf{B}$ and that $\mathbf{A}$ is an involutory matrix, show that the solution is given by $\mathbf{X} = \mathbf{AB}$. [3]

B6. (a) Express $\log_3 36$ in terms of $\log_3 2$. [2]

(b) Find the gradient function for the graph of $y = x^2$ using first principles. [3]

(c) Show that,

$$(1 - \cos\theta)(1 + \sec\theta) \equiv \sec\theta - \cos\theta.$$

[3]

(d) It is given that $f(x) = 3x - 5$ and $g(x) = x^2 + 1$ for $x \geq 0$,

(i) find $f(2)$, [1]

(ii) determine $g^{-1}(x)$, the inverse function of $g(x)$. [2]

(e) Use integration by parts to find $\int_0^1 (x+1)e^x \, dx$, giving the answer in exact form. [4]

(f) Determine the turning points for the graph of $y = x^3 - 6x^2 - 16$ and classify their nature using the second-derivative test. [5]

B7. (a) (i) State the three conditions that should be satisfied for a function $f(x)$ to be continuous at a point $x = a$. [3]

(ii) It is given that,

$$f(x) = \frac{x^2 - 1}{x - 1}.$$

Determine whether $f(x)$ is continuous at $x = 1$. [3]

(b) (a) Evaluate

$$\lim_{x \to 2} \left(x^2 - x + 3\right).$$

[2]

(b) Given that $f(\theta) = \sin^2\theta$, show that $f'(\theta) = \sin(2\theta)$. [2]

(c) Given that $f(x) = (x^2 + 1)e^x$, show that $f'(x) = (x + 1)^2 e^x$. [4]

(d) Show that the equation;

$$\sin(\theta - 30°) = 2\cos(\theta + 60°)$$

can be expressed in the form,

$$\tan\theta = k, \text{ where } k \in \mathbb{R}.$$

Hence, solve the equation $\sin(\theta - 30°) = 2\cos(\theta + 60°)$ for $0° \leq \theta \leq 360°$. [6]

B8. (a) Evaluate the following definite integral using the substitution $u = x^2$,

$$\int_0^1 2xe^{x^2}\,dx.$$

[3]

(b) Determine the values of θ on the interval $-180° \leq \theta \leq 180°$ for which,

$$\sin 2\theta - \cos\theta = 0.$$

[4]

(c) Calculate the volume of the solid generated by rotating the graph of $y = 3x$ about the x-axis for $0 \leq x \leq 2$. [4]

(d) It is given that $f(x) = 2x^4 + ax^3 + bx^2 - 8x + c$, find the values of a, b and c such that the following conditions are satisfied

- $(x+2)^2$ is a factor of $f(x)$.
- $f(x)$ leaves a remainder of 16 when divided by $(x-2)$.

Hence, factorise $f(x)$ and write down the values of x for which $f(x) = 0$. [9]

END OF QUESTION PAPER