NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA1116

FACULTY OF APPLIED SCIENCE

DEPARTMENT OF APPLIED MATHEMATICS

SMA1116: ENGINEERING MATHEMATICS IA

JULY 2023 SPECIAL EXAMINATION

Time : 3 hours

Candidates should attempt all questions from Section A [**40 MARKS**] and ANY THREE questions in Section B [**60 MARKS**].

SECTION A: Answer ALL questions in this section [40]

A1. Define the following terms, giving examples where necessary:

(a) $\epsilon - \delta$ definition of a limit of a function $\mu(x)$, [3]

(b) parallel vectors, [2]

(c) the family of curves in integration, [3]

(d) the argument of a complex number $\mathbf{z}$. [1]

A2. Use the $\epsilon - \delta$ definition to show that,

$$\lim_{x \to 2}(x^2 + x - 2) = 1.$$

[5]

A3. Evaluate:

(a) $\displaystyle\lim_{x \to 1^-} \frac{1}{x-1} + e^{x^2}$, [3]

(b) $\displaystyle\lim_{x \to -3} |x+1| + \frac{3}{x}$. [3]

A4. Find, $\int (\ln \sin \theta)^{\cos \theta} d\theta.$ [5]

A5. Find values of x and y if $z = x + iy$ satisfies $\dfrac{z}{z+2} = 2 - i.$ [5]

A6. (a) Find c if $y = 2x + c$ is tangent to $x^2 + y^2 + 4x - 10y - 7 = 0.$ [5]

(b) Find the distance between the planes $5x + 4y + 3z = 8$ and $5x + 4y + 3z = 1.$ [5]

SECTION B: Answer ANY THREE questions in this section [60]

B7. (a) The region enclosed by the curve $y = x^3$, the $x-$axis and the line $y = 2$ is divided into two equal portions by the line $x = k$. Show that $k = S^{\frac{3}{4}}$, giving the value of S. [3]

(b) If $f(\theta) = 2\sin(\theta^2 + 4)^{\frac{1}{2}}$, find $f'(\theta)$. [5]

(c) Hence evaluate $\displaystyle\int_0^2 \frac{\theta}{\sqrt{\theta^2 + 4}} \cot(\theta^2 + 4)^{\frac{1}{2}} d\theta.$ [6]

(d) Express the roots of $z^7 - 8 - 8i = 0$ in the form $r(\cos\theta + i \sin\theta)$. [6]

B8. (a) Find the distance between the point $P(2, 3, 1)$ and the line $\vec{r} = (-1, 0, 2) + t(5, 1, 2)$. [5]

(b) Use De Moivre's theorem to express $\sin^3\theta$ in terms of $\sin\theta$. [7]

(c) Grains of wheat leaking from a sack on a broken conveyor belt, at a rate of $9\ m^3$ per minute, form a heap on the ground, in the shape of a right circular cone with a vertical angle of 60^0. Show that in ten seconds, the radius of the base of the cone increases at $3^{\frac{1}{2}}(\frac{4}{\pi})^{\frac{1}{3}}$ m per minute. [8]

B9. (a) The length of a rectangle is twice the breadth. If the area of the rectangle increases by 5%, by what percentage will the perimeter increase? [5]

(b) Find the Cartesian equations of the plane that contains the point A(2,2,1) and the line line

$$\vec{r} = (4i - 2j + 2k) + t(i + 2j + k),$$

[7]

(c) Use the substitution $x \sec^2\theta = \tan^2\theta - 2$ to show that,

$$\int_{-\frac{1}{2}}^{1} \sqrt{\frac{1-x}{2+x}}\, dx = \frac{3}{4}(\pi - 2).$$

[8]

B10. (a) Show that at the point $P\Big(y(t-1), t^2+t\Big)$ the gradient of the tangent of a curve which passes through this point is

$$\frac{(2x+3y)y}{3x^2+6xy+2y^2}.$$

[5]

(b) Find the distance between the lines

$$\vec{r}\,(t) = (2-t, 1+t, 4)$$

and

$$\vec{r}\,(s) = (5s-1, s, 2+2s).$$

[3]

(c) Find the constants a, b and c if the line

$$\vec{r}\,(t) = (4+2t, 2-t, -1-2t),$$

lies on the plane $2x + by + cz = 1$. [4]

(d) A student draws two tangents at two points on the curve,

$$\frac{1}{2}(x+y)^2 = 12 - (x-y)^2$$

and finds that at one point, the tangent is parallel to the $x-$axis and to the $y-$axis at the other point. Find the coordinates of these points. [8]

END OF QUESTION PAPER