NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA1116

FACULTY OF APPLIED SCIENCE

DEPARTMENT OF APPLIED MATHEMATICS

SMA1116: ENGINEERING MATHEMATICS IA

DECEMBER 2024 EXAMINATION

Time : 3 hours

Candidates should attempt all questions from Section A [**40 MARKS**] and ANY THREE questions in Section B [**60 MARKS**].

SECTION A: Answer ALL questions in this section [40]

A1. If the plane $\pi : 2x + by + cz = 1$ contains the line $\vec{r}\,(t) = (4 + 2t, 2 - t, -1 - 2t)$.

(a) Find the values of b and c. [4]

(b) Find the shortest distance of the point P with position vector $2\mathbf{j} + 4\mathbf{k}$ from the line above. [4]

A2. Show that if the gradient to the curve $y = \sqrt{\tan x}$ is equal to 1 at the point where $x = a$, then $\tan^3 a + \tan^2 a + 3\tan a - 1 = 0$. [5]

A3. Given that $\lim_{x \to 5^-} \dfrac{x^2 + kx - 20}{x - 5} = \lim_{x \to 5^+} \dfrac{x^2 + kx - 20}{x - 5}$, find the value of k and hence evaluate the limit. [6]

A4. Solve the equation $(z+2+i)^*+(2+i)z=0$, where $*$ represents the conjugate, giving your answer in the form $x+iy$. [5]

A5. Evaluate the following limits

(i) $\lim_{x\to 1}\dfrac{(x^2-1)}{|(x-1)|}$ [3]

(ii) $\lim_{x\to 3^-}\dfrac{x^2|(x-3)|}{(x-3)}$. [3]

A6. Find the coordinates of the points on the curve $x^3+3xy^2-y^3=5$ where the gradient is undefined. [5]

A7. Given that the real number p is such that $\frac{\pi}{4}\le arg(w+p)\le\frac{3\pi}{4}$, where

$$w=\frac{22+4i}{(2-i)^2},$$

find the possible set of values of p. [5]

SECTION B: Answer ANY THREE questions in this section [60]

B8. (a) Find the value of the constant a if the lines $\vec{r}=a\mathbf{i}+2\mathbf{j}+3\mathbf{k}+\lambda(\mathbf{i}-2\mathbf{j}+3\mathbf{k})$ and $\vec{r}=2\mathbf{i}+\mathbf{j}+2\mathbf{k}+\mu(2\mathbf{i}-\mathbf{j}+\mathbf{k})$ intersect. [3]

(b) Hence find the equation of the plane which contains both lines. [5]

(c) The complex numbers u and v satisfy $v+iw=5$ and $(1+2i)v-w=3i$. Express u and v in the form $x+iy$, x and y real. [4]

(d) Calculate the least value of arg z for points on the locus $|z-2-3i|=1$. [3]

(e) Given that the variables x and y satisfy the relation $\sin y\cos x=\sin x$, show that

$$\frac{dy}{dx}=\frac{\sec x}{\sqrt{(\cos 2x)}}.$$

[5]

B9. (a) Find the greatest value of the argument of z for points in the region $|z - 3i| \leq 1$ and Re $z \leq 0$. [3]

(b) The line l has the equation $(5 + \lambda)\mathbf{i} + (-3 - 2\lambda)\mathbf{j} + (-1 + \lambda)\mathbf{k}$ and the plane p has the equation $(\vec{r} - \mathbf{i} - 2\mathbf{j}).(3\mathbf{i} + \mathbf{j} + \mathbf{k}) = 0$.

(i) Find the position vector of the point of intersection of the line and the plane. [3]

(ii) Find the equation of the line which lies in p and intersects the line l at right angles. [5]

(c) Find the x-coordinate of the maximum point of the curve $y = e^{\cos x} \sin^3 x$ in the interval $0 \leq x \leq \pi$. [4]

(d) Evaluate

$$\int_0^{\pi} e^{\sin(\frac{\pi}{2} - x)} \sin^3 x dx.$$

[5]

B10. (a) Let $u = 3 - i$ and u^* be its conjugate, show that $\tan^{-1}(a) = 2\tan^{-1}(b)$ where a and b are rational numbers to be found. [4]

(b) The two tangents to the curve $x^3 + y^3 + 2xy + 8 = 0$ at the points where $x = 0$ and $y = 0$ meet at an angle α. Find the exact value of $\tan \alpha$. [5]

(c) The line l has the equation $\vec{r} = 4\mathbf{i} + 3\mathbf{j} - \mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$ and the plane p has the equation $2x - 3y - z = 4$.

Find the equation of a second plane q which is parallel to l, perpendicular to p and contains the point with position vector $4\mathbf{j} - \mathbf{k}$. [6]

(d) Find the area enclosed by the lines $x = 0$, $x = \frac{\pi}{2}$, the x-axis and the curve $y = \sin^3 x\sqrt{\cos x}$. [5]

B11. (a) Given that the position vectors of points A, B and C are $-2\mathbf{i} + \mathbf{j} + 4\mathbf{k}, 5\mathbf{i} + 2\mathbf{j}$ and $8\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$ respectively.

(i) Find the vector equation of the line l which passes through points B and C. [2]

(ii) A second line l_2 given by $\vec{r} = -2\mathbf{i} + \mathbf{j} + 4\mathbf{k} + \mu(3\mathbf{i} + \mathbf{j} - 2\mathbf{k})$. Find the coordinates of the point of intersection of the two lines. [3]

(iii) The point D on l_2 is such that $AB = BD$, find the position vector of D. [8]

(b) The variables x and θ satisfy the equation

$$(x^2 + 9)\sin\theta \frac{d\theta}{dx} = (x + 3)\cos^4\theta,$$

so that when $\theta = \frac{\pi}{3}, x = 3$. Find the value of $\cos\theta$ when $x = 0$. [7]

B12. (a) On a sketch of an Argand diagram, shade the region whose points represent the complex numbers z satisfying the inequalities $|z-4-2i| \leq 3$ and $|z| \geq |10-z|$. [3]

(b) Find the greatest value of arg (z) for points in this region. [2]

(c) The variables θ and y are such that

$$(1 + y)(1 + \cos 2\theta)\frac{dy}{d\theta} = e^{3y},$$

and $y = 0$ when $\theta = \frac{\pi}{4}$. Find the value of $\tan\theta$ when $y = 1$. [5]

(d) Show that $\cos^4\theta - \sin^4\theta \equiv 2\cos^2\theta - 1 \equiv 1 - 2\sin^2\theta$ [4]

Hence evaluate

$$\int_{-\frac{\pi}{8}}^{\frac{\pi}{8}} (\cos^4\theta - \sin^4\theta + 4\sin^2\theta\cos^2\theta)d\theta.$$

[6]

END OF QUESTION PAPER