NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA1201

FACULTY OF APPLIED SCIENCE

DEPARTMENT OF APPLIED MATHEMATICS

SMA1201: CALCULUS OF SEVERAL VARIABLES

SEPTEMBER 2024 EXAMINATION

Time : 3 hours

Candidates should attempt all questions from Section A **[40 MARKS]** and ANY THREE questions in Section B **[60 MARKS].**

SECTION A: ATTEMPT ALL QUESTIONS.

A1. (a) Define the domain of a function of several variables. [1]

(b) Determine and sketch in $\Re^2$ the domain of the function $\ln(9 - x^2 - 9y^2)$. [3]

(c) Sketch and name the surface in three dimensional space represented by the following;

(i) $2z + y = 2$. [2]

(ii) $x^2 + y^2 = 4$. [3]

A2. (a) Determine whether or not the following limit exists

$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$. [3]

(b) Using the ϵ-δ definition of a limit, prove that $\lim_{(x,y)\to(1,2)}(3x + 2y) = 7$. [4]

A3. (a) Find a potential function for the vector field,

$$\vec{F} = 2xy^3z^4\vec{i} + 3x^2y^2z^4\vec{j} + 4x^2y^3z^3\vec{k}$$

[6]

A4. (a) Find the equation of the tangent plane and normal line at the point $(4, -1, 1)$ to the surface $x^2 + 2y^2 + 3z^2 = 21$. [4]

(b) For $f(x, y) = xe^{x^2y}$, Find f_x and f_y and evaluate each at the point $(1, ln2)$. [3]

(c) Find the relative extrema of the function $f(x, y) = -x^3 + 4xy - 2y^2 + 1$. [4]

A5. (a) Given a region of integration **R** of $\int_0^1 \int_1^{e^y} f(x, y)dxdy$

(i) Sketch the region [1]

(ii) Change the order of integration [2]

(b) Find the the integral of

$$\int_0^1 \int_x^1 e^{y^2} dydx$$

[4]

SECTION B: [60].

B6. (a) Find the volume of the solid bounded by the curves $z = \sqrt{x^2 + y^2}$ and $x^2 + y^2 + z^2 = 9$ in the first quadrant. [8]

(b) Evaluate $\int\int\int_E 4xy \text{ dV}$ where E is the region bounded by $z = 2x^2 + 2y^2 - 7$ and $z = 1$. [7]

(c) Evaluate $\int\int_D 2yx^2 + 9y^3 \text{dA}$ where D is the region bounded by $y = \frac{2}{3}x$ and $y = 2\sqrt{x}$. [5]

B7. (a) Using Lagrange multipliers, find the maximum and minimum values of $f(x, y) = 81x^2 + y^2$ subject to constraint $4x^2 + y^2 = 9$. [12]

(b) Sketch the graph of $z = \frac{x^2}{4} + \frac{y^2}{4} - 6$. [8]

B8. (a) For the space given by:

$$\vec{r}(t) = t\vec{i} + 3sin(t)\vec{j} + 3cos(t)\vec{k}$$

Find:

(i) the unit tangent vector $\vec{T}(t)$, [3]

(ii) the unit normal vector $\vec{N}(t)$, [4]

(iii) the bi-normal vector $\vec{B}(t)$, [4]

(iv) the curvature κ. [4]

(b) Determine the arc length function for $\vec{r}(t) = 2t\vec{i} + 3sin(2t)\vec{j} + 3cos(2t)\vec{k}$ on the interval $0 \leq t \leq 2\pi$. [5]

B9. (a) Use Green's Theorem to evaluate $\int_C yx^2dx - x^2dy$ where C is **positively oriented** and encloses the **left half** of a circle of radius 5 centred at the origin. [12]

(b) Evaluate $\int_C \nabla f \cdot d\vec{r}$ where $f(x, y, z) = \cos(\pi x) + \sin(\pi y) - xyz$ and C is the line path that starts at $\left(1, \frac{1}{2}, 2\right)$ and ends at $(2, 1, -1)$. [4]

(c) Calculate div$\vec{F}$ and curl$\vec{F}$ for $\vec{F} = x^2y\vec{i} - (z^3 - 3x)\,\vec{j} + 4y^2\vec{k}$ [4]

END OF QUESTION PAPER